LESSON MAKER PRO

Close to the Point

Estimate limits from expressions, tables, and graphs.

HIGH SCHOOL · F.IF.C.8.d

F.IF.C.8.d "Estimate limits algebraically, numerically and graphically."

NAME _____ DATE _____

FINDING A LIMIT

A limit tells you what value a function approaches as x gets close to a number. You can simplify an expression, check values from both sides in a table, or follow a graph toward that number. A two-sided limit exists only when the values from the left and right approach the same result.

WORKED EXAMPLE

Find the limit of $(x^2 - 49)/(x - 7)$ as x approaches 7.

1. Factor the numerator: $x^2 - 49 = (x - 7)(x + 7)$.
2. For $x \neq 7$, cancel $x - 7$ to get $x + 7$.
3. As x approaches 7, $x + 7$ approaches 14.

Answer: 14

PRACTICE 6 problems

1 Find the limit of $(x^2 - 4)/(x - 2)$ as x approaches 2.

ANSWER _____

2 Find the limit of $(x^2 + 3x + 2)/(x + 1)$ as x approaches -1.

ANSWER _____

3 Use the table to estimate the limit of f(x) as x approaches 3.

VALUES OF F(X) NEAR 3

x	f(x)
2.9	5.9
2.99	5.99
3.01	6.01
3.1	6.1

ANSWER _____

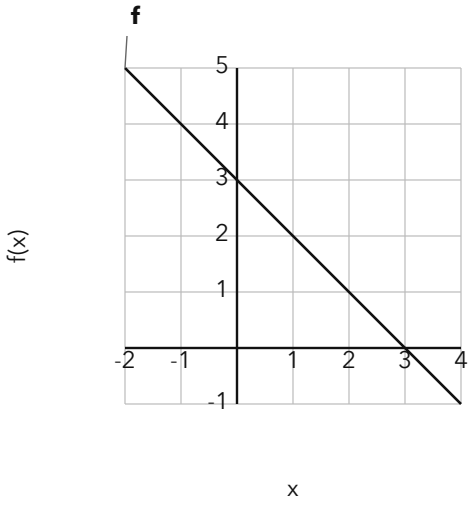
4 Use the table to estimate what f(x) approaches as x approaches 0 from each side. Does the two-sided limit exist?

VALUES OF F(X) NEAR 0

x	f(x)
-0.1	-1.1
-0.01	-1.01
0.01	1.01
0.1	1.1

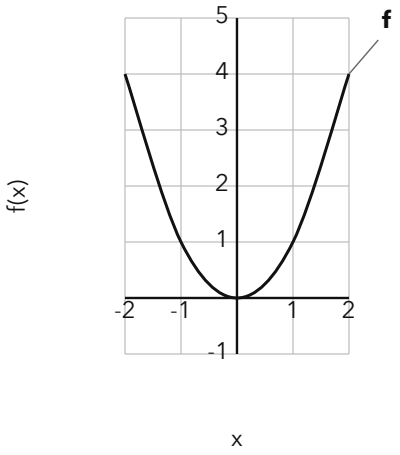
ANSWER _____

5 Use the graph to estimate the limit of f(x) as x approaches 1.



ANSWER _____

6 Use the graph to estimate the limit of f(x) as x approaches 0.



ANSWER _____

APPLY IT Show your work

1 A streamer records $D(x) = x^2 + 6x$ megabytes of downloaded data after x seconds, where $x > 0$. The average download rate is $D(x)/x$. Estimate the average rate as x approaches 0 from the right.

WORK SPACE

ANSWER _____

2 A phone's processor temperature, in °C, is modeled by $T(x) = 30 + (x^2 - 1)/(x - 1)$ near $x = 1$, where x is minutes after an app opens and $x \neq 1$. Estimate the temperature as x approaches 1.

WORK SPACE

ANSWER _____

CHALLENGE One step up

For $x < 2$, $f(x) = (x^2 - 4)/(x - 2)$. For $x > 2$, $f(x) = x + 3$. Does the limit of f(x) as x approaches 2 exist? Explain using both one-sided limits.

WORK SPACE AND ANSWER

ANSWER KEY # Close to the Point

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the limit of $(x^2 - 4)/(x - 2)$ as x approaches 2.	**4**
2	Find the limit of $(x^2 + 3x + 2)/(x + 1)$ as x approaches -1.	**1**
3	Use the table to estimate the limit of f(x) as x approaches 3.	**6**
4	Use the table to estimate what f(x) approaches as x approaches 0 from each side. Does the two-sided limit exist?	**From the left: -1. From the right: 1. The two-sided limit does not exist.**
5	Use the graph to estimate the limit of f(x) as x approaches 1.	**2**
6	Use the graph to estimate the limit of f(x) as x approaches 0.	**0**

PART 2 · APPLY IT

1. **6 megabytes per second** For $x > 0$, $D(x)/x = x + 6$. As x approaches 0 from the right, the average rate approaches 6 megabytes per second.
2. **32°C** For $x \neq 1$, $(x^2 - 1)/(x - 1) = x + 1$. Thus T(x) approaches 30 + 1 + 1 = 32°C.

CHALLENGE

No. From the left, f(x) = x + 2 approaches 4. From the right, f(x) = x + 3 approaches 5. Because the one-sided limits differ, the two-sided limit does not exist.

Accept DNE for a limit that does not exist. In the graph problems, students should read the value the graph approaches, whether or not the function has a value at that x.