LESSON MAKER PRO

Growth Has Limits

What do the numbers in a logistic model tell you?

HIGH SCHOOL · F.LE.B.5.a

F.LE.B.5.a "Interpret the parameters in a logistic function in terms of a context."

NAME _____ DATE _____

READ THE PARAMETERS

In $P(t) = L/(1 + 2^{h-t})$, L is the value P(t) approaches over time. At t = h, the denominator is 2, so P(h) = L/2.

WORKED EXAMPLE

A game models active players with $P(t) = 72/(1 + 2^{3-t})$, where t is weeks after launch. Explain what 72 and 3 mean, then find P(4).

ACTIVE PLAYERS

Week t	P(t)
3	36
4	?

1. 72 is the value P(t) approaches.
2. At week 3, P(3) = 72/(1 + 1) = 36.
3. At week 4, P(4) = 72/(1 + 1/2) = 48.

Answer: The model approaches 72 active players. It reaches half that number at week 3. At week 4, it predicts 48 active players.

PRACTICE 4 problems

1 A club models membership with $P(t) = 120/(1 + 2^{4-t})$, where t is months. What do 120 and 4 mean?

ANSWER _____

2 An app models users with $P(t) = 96/(1 + 2^{2-t})$, where t is months. When does it reach half its long-run limit?

ANSWER _____

3 A team models sign-ups with $P(t) = 180/(1 + 2^{5-t})$, where t is weeks. How many sign-ups does it predict at week 5?

ANSWER _____

4 A site models subscribers with $P(t) = 150/(1 + 2^{1-t})$, where t is months. What does 150 mean, and how many subscribers does the model predict at month 2?

ANSWER _____

APPLY IT Show your work

1 A music playlist models weekly listeners with $P(t) = 240/(1 + 2^{4-t})$, where t is weeks since its release. What does 4 mean? How many listeners does the model predict at week 5?

WORK SPACE

ANSWER _____

2 A school models lunch preorders with $P(t) = 600/(1 + 3^{2-t})$, where t is weeks after ordering opens. What do 600 and 2 mean? How many preorders does the model predict at week 3?

WORK SPACE

ANSWER _____

CHALLENGE One step up

An esports club uses $P(t) = L/(1 + 3^{h-t})$ to model attendance t weeks after sign-ups open. Use the table to find L and h. Explain what each means.

ESPORTS CLUB ATTENDANCE

Week t	P(t)
4	25
5	50
6	75

WORK SPACE AND ANSWER

ANSWER KEY

Growth Has Limits

HIGH SCHOOL · F.LE.B.5.a

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	A club models membership with $P(t) = 120/(1 + 2^{4-t})$, where t is months. What do 120 and 4 mean?	**Membership approaches 120 members and reaches 60 members at month 4.**
2	An app models users with $P(t) = 96/(1 + 2^{2-t})$, where t is months. When does it reach half its long-run limit?	**Month 2.**
3	A team models sign-ups with $P(t) = 180/(1 + 2^{5-t})$, where t is weeks. How many sign-ups does it predict at week 5?	**90 sign-ups.**
4	A site models subscribers with $P(t) = 150/(1 + 2^{1-t})$, where t is months. What does 150 mean, and how many subscribers does the model predict at month 2?	**Subscribers approach 150. At month 2, the model predicts 100 subscribers.**

PART 2 · APPLY IT

1. **The playlist reaches half its long-run limit, 120 listeners, at week 4. It predicts 160 listeners at week 5.** The parameter 4 gives the week when $P(t) = 240/2$. Substitute $t = 5$ to get $240/(1 + 1/2) = 160$.
2. **Preorders approach 600 and reach 300 at week 2. The model predicts 450 preorders at week 3.** At $t = 2$, the denominator is 2. At $t = 3$, it is $1 + 1/3$, so $P(3) = 600/(4/3) = 450$.

CHALLENGE

L = 100 and h = 5. Attendance approaches 100 students and reaches half that number, 50 students, at week 5.

Accept wording such as "long-run limit" for the value approached. The model approaches L but does not reach it at a finite time.