LESSON MAKER PRO

Where Curves Level

Read, analyze, and sketch logistic graphs

HIGH SCHOOL · F.LE.B.6.a

F.LE.B.6.a "Sketch and analyze the graphs of logistic functions."

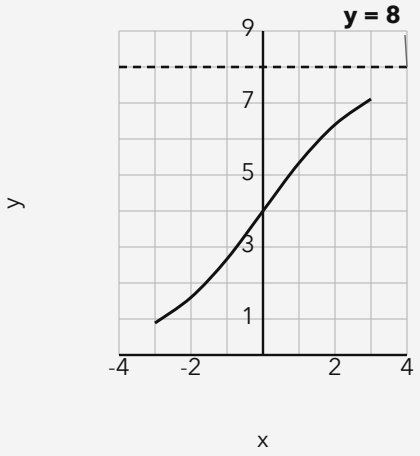
NAME _____ DATE _____

THE BIG IDEA

A logistic graph rises in an S shape and levels off near two horizontal lines called asymptotes. For $y = K/(1 + a \cdot b^{-x})$, where K and a are positive and $b > 1$, those lines are $y = 0$ and $y = K$. The midpoint has a height of K/2.

WORKED EXAMPLE

Use the graph to find the upper asymptote and the midpoint.



1. Read the dashed line: the upper asymptote is $y = 8$.
2. Halfway between $y = 0$ and $y = 8$ is $y = 4$. The curve reaches it at $x = 0$.

Answer: Upper asymptote: y = 8. Midpoint: (0, 4).

PRACTICE 4 problems

1 For $f(x) = 12/(1 + 3 \cdot 2^{-x})$, find the upper asymptote and f(0).

ANSWER _____

2 For $g(x) = 10/(1 + 2^{-(x-1)})$, find the midpoint and upper asymptote.

ANSWER _____

3 Find both horizontal asymptotes of $h(x) = 6/(1 + 2^{-x})$.

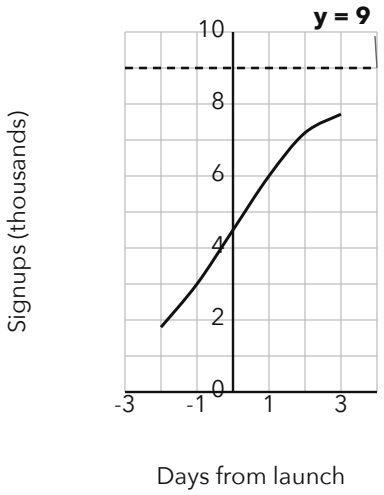
ANSWER _____

4 For $j(x) = 14/(1 + 2^{-(x-2)})$, find the midpoint. Does the graph rise or fall?

ANSWER _____

APPLY IT Show your work

1 A game's beta signups are shown by days from launch. How many signups does the graph show on launch day, and what total does it approach?



ANSWER _____

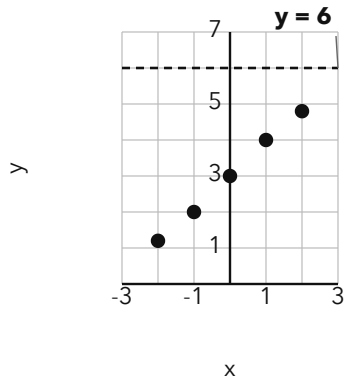
2 A student-run app models its users, in hundreds, by $U(x) = 20/(1 + 3^{-(x-1)})$, where x is weeks after launch. When does it reach half its long-run user count, and how many users is that?

WORK SPACE

ANSWER _____

CHALLENGE One step up

The points follow a logistic model. Sketch its curve through the points. Mark both horizontal asymptotes and its midpoint.



WORK SPACE AND ANSWER

ANSWER KEY

Where Curves Level

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For $f(x) = 12/(1 + 3 \cdot 2^{-x})$, find the upper asymptote and f(0).	**Upper asymptote: y = 12. f(0) = 3.**
2	For $g(x) = 10/(1 + 2^{-(x-1)})$, find the midpoint and upper asymptote.	**Midpoint: (1, 5). Upper asymptote: y = 10.**
3	Find both horizontal asymptotes of $h(x) = 6/(1 + 2^{-x})$.	**y = 0 and y = 6.**
4	For $j(x) = 14/(1 + 2^{-(x-2)})$, find the midpoint. Does the graph rise or fall?	**Midpoint: (2, 7). The graph rises.**

PART 2 · APPLY IT

1. **4,500 signups on launch day; the total approaches 9,000 signups.** At day 0, the graph shows 4.5 thousand. Its upper asymptote is 9 thousand.
2. **After 1 week; 1,000 users.** The model approaches 20 hundred users. At x = 1, U(1) = 10 hundred, which is half that amount.

CHALLENGE

An increasing S-curve through the points, approaching y = 0 on the left and y = 6 on the right. Horizontal asymptotes: y = 0 and y = 6. Midpoint: (0, 3).

Accept a smooth, increasing sketch that passes through the plotted points and levels toward both asymptotes. In the word problems, accept answers in either users or hundreds of users when units are clear.