LESSON MAKER PRO

Growth Model Matchup

Compare how three models grow.

HIGH SCHOOL · F.LE.B.6.b

F.LE.B.6.b "Compare and contrast the exponential, logarithmic, and logistic models."

NAME _____ DATE _____

SPOT THE PATTERN

For equally spaced inputs, an exponential model has a constant output ratio. A logarithmic model adds a constant amount whenever its positive inputs are multiplied by the same factor. A logistic model rises toward a fixed upper limit, with gains that eventually shrink.

WORKED EXAMPLE

Classify the model shown in the table and write a rule for its output y.

MODEL VALUES

x	y
5	11
25	22
125	33
625	44

1. Each x-value is 5 times the one before it.
2. Each y-value is 11 more than the one before it.
3. Since $\log_5(5)=1$, multiply $\log_5(x)$ by 11 to match the table.

Answer: Logarithmic; $y=11\log_5(x)$.

PRACTICE 6 problems

1 Classify the model in the table. What is its output multiplier for each increase of 1 in x?

MODEL A

x	y
0	2
1	4
2	8
3	16

ANSWER _____

_____

2 Classify the model in the table. If the pattern continues, what is y when x=16?

MODEL B

x	y
1	0
2	1
4	2
8	3

ANSWER _____

_____

3 Which model type best fits the table: exponential, logarithmic, or logistic? Give one numerical clue.

MODEL C

x	y
-2	6
-1	10
0	15
1	20
2	24

ANSWER _____

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4 Let $E(x)=2^x$ and $L(x)=\log_2(x)$. At x=4, which output is larger, and by how much?

ANSWER _____

_____

5 Let $E(x)=4^x$ and $L(x)=\log_4(x)$. What input gives an output of 64 under E? What input gives an output of 3 under L?

ANSWER _____

_____

6 Let $E(x)=6(3/2)^x$ and $S(x)=18/(1+2(1/2)^x)$. At x=2, which output is greater, and by how much?

ANSWER _____

_____

APPLY IT Show your work

1 A campus has 1,200 students. Two models predict downloads of a new app after t weeks: $E(t)=400(2)^t$ and $S(t)=1200/(1+2(1/2)^t)$. Find both predictions at t=2. Which model respects the campus limit?

WORK SPACE

ANSWER _____

2 Two models predict views of a gaming clip after t days: $E(t)=50(2)^t$ and $L(t)=100\log_2(t)$, where t is at least 1. Find both predictions on day 4. Which model gains exactly 100 views each time t doubles?

WORK SPACE

ANSWER _____

CHALLENGE One step up

For integer inputs x at least 1, let $E(x)=2^x$, $L(x)=2\log_2(x)$, and $S(x)=16/(1+(1/2)^x)$. Find the smallest x for which E(x) exceeds both other outputs. Explain why no smaller allowed input works.

WORK SPACE AND ANSWER

ANSWER KEY

Growth Model Matchup

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Classify the model in the table. What is its output multiplier for each increase of 1 in x?	**Exponential; the output multiplier is 2.**
2	Classify the model in the table. If the pattern continues, what is y when x=16?	**Logarithmic; y=4.**
3	Which model type best fits the table: exponential, logarithmic, or logistic? Give one numerical clue.	**Logistic; the gains are 4, 5, 5, and 4, so the gains begin to shrink.**
4	Let $E(x)=2^x$ and $L(x)=\log_2(x)$. At x=4, which output is larger, and by how much?	**E(4)=16 is larger than L(4)=2 by 14.**
5	Let $E(x)=4^x$ and $L(x)=\log_4(x)$. What input gives an output of 64 under E? What input gives an output of 3 under L?	**E(3)=64, so its input is 3. L(64)=3, so its input is 64.**
6	Let $E(x)=6(3/2)^x$ and $S(x)=18/(1+2(1/2)^x)$. At x=2, which output is greater, and by how much?	**E(2)=27/2 is greater than S(2)=12 by 3/2.**

PART 2 · APPLY IT

1. **E(2)=1,600 downloads and S(2)=800 downloads. The logistic model S respects the 1,200-student limit.** Substitute t=2 into both rules. E predicts more downloads than the number of students, while S stays below 1,200.
2. **E(4)=800 views and L(4)=200 views. The logarithmic model L gains 100 views each time t doubles.** Compute $50(2)^4=800$ and $100\log_2(4)=200$. Doubling t raises $\log_2(t)$ by 1, so L rises by 100.

CHALLENGE

x=4. Then E(4)=16, L(4)=4, and S(4)=256/17, which is less than 16. For x=1, 2, or 3, E(x) is at most 8, while S(x) is at least S(1)=32/3, which is greater than 8.

Accept equivalent fraction or decimal forms for exact values. For Problem 3, accept a numerical explanation that the gains rise and then begin to fall.