LESSON MAKER PRO

When Will It Happen?

Use logarithms to find times in growth models.

HIGH SCHOOL · F.LE.B.6.c

F.LE.B.6.c "Apply understanding of logarithmic and logistic functions to solve real-world problems."

NAME _____ DATE _____

FIND THE TIME

A logarithm undoes an exponent: if $b^x = k$, then $x = \log_b(k)$. In a logistic model $P(x) = L/(1 + a \cdot b^x)$, L is the upper limit. To find when the model reaches a target, set P(x) equal to the target and isolate b^x.

WORKED EXAMPLE

A music site's monthly listeners, in thousands, follow $M(x) = 150/(1 + 27(1/3)^x)$, where x is months. When does the model reach 75 thousand listeners?

1. Set the model equal to 75: $75 = 150/(1 + 27(1/3)^x)$.
2. Divide 150 by 75: $1 + 27(1/3)^x = 2$.
3. Subtract 1, then divide by 27: $(1/3)^x = 1/27$.
4. Since $(1/3)^3 = 1/27$, $x = 3$.

Answer: 3 months

PRACTICE 6 problems

1 Solve $9 \cdot 2^x = 144$.

ANSWER _____

2 Solve $\log_5(x) = 2$.

ANSWER _____

3 Solve $11 \cdot 10^x = 1100$.

ANSWER _____

4 For $P(x) = 80/(1 + 16(1/2)^x)$, find x when P(x) = 40.

ANSWER _____

5 For $P(x) = 90/(1 + 49(1/7)^x)$, find x when P(x) = 45.

ANSWER _____

6 For $P(x) = 84/(1 + 8(1/2)^x)$, find x when P(x) = 56.

ANSWER _____

APPLY IT Show your work

1 An independent game gets 18 downloads on its first day. Its total downloads double each day, following $D(x) = 18 \cdot 2^x$, where x is days after the first day. When does the model predict 576 total downloads?

WORK SPACE

ANSWER _____

2 A school concert can sell at most 180 tickets. A model for tickets sold x weeks after sales begin is $T(x) = 180/(1 + 8(1/2)^x)$. When does the model predict 90 tickets sold?

WORK SPACE

ANSWER _____

CHALLENGE One step up

An app's weekly active users, in thousands, follow $U(x) = 160/(1 + 16(1/2)^x)$, where x is weeks. How many weeks after the model reaches half its upper limit does it reach 80% of that limit? Show how you found both times.

WORK SPACE AND ANSWER

ANSWER KEY

When Will It Happen?

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Solve $9 \cdot 2^x = 144$.	**x = 4**
2	Solve $\log_5(x) = 2$.	**x = 25**
3	Solve $11 \cdot 10^x = 1100$.	**x = 2**
4	For $P(x) = 80/(1 + 16(1/2)^x)$, find x when P(x) = 40.	**x = 4**
5	For $P(x) = 90/(1 + 49(1/7)^x)$, find x when P(x) = 45.	**x = 2**
6	For $P(x) = 84/(1 + 8(1/2)^x)$, find x when P(x) = 56.	**x = 4**

PART 2 · APPLY IT

1. **5 days after the first day** Set $18 \cdot 2^x = 576$, so $2^x = 32$. Thus x = 5.
2. **3 weeks after sales begin** Set T(x) = 90 and simplify to $8(1/2)^x = 1$. Then $(1/2)^x = 1/8$, so x = 3.

CHALLENGE

Half the limit is 80, reached at x = 4 because $16(1/2)^x = 1$. Eighty percent of the limit is 128, reached at x = 6 because $16(1/2)^x = 1/4$. The difference is 2 weeks.

Accept equivalent logarithmic forms for the times. For the challenge, look for both times and their difference.