LESSON MAKER PRO

Growth With Limits

Model how sign-ups grow toward a maximum

HIGH SCHOOL · F.LE.B.6

F.LE.B.6 "Build and interpret logistic functions to model real-world problems."

NAME _____ DATE _____

THE LOGISTIC RULE

A logistic model has the form $P(t) = K/(1 + ab^t)$, where $0 < b < 1$ and K is the level P(t) approaches. If you know K and the values at $t = 0$ and $t = 1$, find $a = K/P(0) - 1$, then find $b = (K/P(1) - 1)/a$.

WORKED EXAMPLE

A club can have at most 80 members. It starts with 20 members and has 32 after one month. Build a logistic model for its membership P(t), where t is months since it started.

1. Use K = 80 and P(0) = 20: $a = 80/20 - 1 = 3$.
2. Use P(1) = 32: $b = (80/32 - 1)/3 = 1/2$.
3. Substitute K, a, and b into $P(t) = K/(1 + ab^t)$.

Answer: $P(t) = 80/(1 + 3(1/2)^t)$

PRACTICE 6 problems

1 A new school club expects its membership to approach 120. It starts with 30 members, and b = 1/3. Write a logistic model P(t), where t is weeks since the club started.

ANSWER _____

2 A video channel expects its subscriber count to approach 150. It has 50 subscribers at t = 0 and 75 at t = 1. Write a logistic model P(t).

ANSWER _____

3 For $P(t) = 100/(1 + 4(1/2)^t)$, find P(2).

ANSWER _____

4 For $P(t) = 240/(1 + 5(1/3)^t)$, find P(0) and the level P(t) approaches.

ANSWER _____

5 For $P(t) = 180/(1 + 9(1/3)^t)$, at what whole-number time t does P(t) reach half of its upper limit?

ANSWER _____

6 For $P(t) = 128/(1 + 8(1/2)^t)$, at what whole-number week does P(t) first reach at least 2/3 of its upper limit?

ANSWER _____

APPLY IT Show your work

1 Students launch a school event app. They expect at most 300 students to sign up. There are 60 sign-ups at week 0 and 100 at week 1. Build a logistic model P(t) for sign-ups, then predict the number at week 2.

WORK SPACE

ANSWER _____

2 An esports tournament has room for 250 players. There are 50 sign-ups at day 0 and 125 at day 1. Build a logistic model P(t) for sign-ups, then predict the number at day 2.

WORK SPACE

ANSWER _____

CHALLENGE One step up

A gaming club records its membership in the table. Assume $P(t) = K/(1 + a(1/2)^t)$, where t is weeks since the first count. Use the counts at weeks 0 and 1 to find K and write the model. Then check whether the model matches the count at week 2. Show your reasoning.

GAMING CLUB MEMBERSHIP

Week t	Members P(t)
0	10
1	18
2	30

WORK SPACE AND ANSWER

ANSWER KEY

Growth With Limits

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	A new school club expects its membership to approach 120. It starts with 30 members, and b = 1/3. Write a logistic model P(t), where t is weeks since the club started.	**$P(t) = 120/(1 + 3(1/3)^t)$**
2	A video channel expects its subscriber count to approach 150. It has 50 subscribers at t = 0 and 75 at t = 1. Write a logistic model P(t).	**$P(t) = 150/(1 + 2(1/2)^t)$**
3	For $P(t) = 100/(1 + 4(1/2)^t)$, find P(2).	**50**
4	For $P(t) = 240/(1 + 5(1/3)^t)$, find P(0) and the level P(t) approaches.	**P(0) = 40; P(t) approaches 240.**
5	For $P(t) = 180/(1 + 9(1/3)^t)$, at what whole-number time t does P(t) reach half of its upper limit?	**t = 2**
6	For $P(t) = 128/(1 + 8(1/2)^t)$, at what whole-number week does P(t) first reach at least 2/3 of its upper limit?	**Week 4**

PART 2 · APPLY IT

1. **$P(t) = 300/(1 + 4(1/2)^t)$; P(2) = 150 sign-ups.** From the two counts, a = 300/60 - 1 = 4 and b = (300/100 - 1)/4 = 1/2. Substitute t = 2 to get 150.
2. **$P(t) = 250/(1 + 4(1/4)^t)$; P(2) = 200 sign-ups.** Here a = 250/50 - 1 = 4 and b = (250/125 - 1)/4 = 1/4. At t = 2, the denominator is 1 + 4/16 = 5/4, so P(2) = 200.

CHALLENGE

K/10 - 1 = 2(K/18 - 1), so K = 90. Then a = 90/10 - 1 = 8, giving $P(t) = 90/(1 + 8(1/2)^t)$. At t = 2, P(2) = 90/(1 + 2) = 30, which matches the table.

Accept equivalent logistic forms with simplified or unsimplified fractions. In Problem 6, the model increases over time and first equals 2/3 of 128 at week 4.