LESSON MAKER PRO

Identity Detective

Rewrite trig expressions, then find exact solutions.

HIGH SCHOOL · F.TF.C.9.a

F.TF.C.9.a "Use trigonometric identities to rewrite expressions and as a tool when solving trigonometric equations."

NAME _____ DATE _____

REWRITE BEFORE YOU SOLVE

The identity $\sin^2 x + \cos^2 x = 1$ lets you replace $\sin^2 x$ with $1 - \cos^2 x$, or replace $\cos^2 x$ with $1 - \sin^2 x$. When solving an equation, factor the new expression and check all angles in the given interval.

WORKED EXAMPLE

Solve $8\sin^2 x + 8\cos x = 8$ for $0 \le x < 2\pi$.

1. Divide by 8: $\sin^2 x + \cos x = 1$.
2. Replace $\sin^2 x$: $1 - \cos^2 x + \cos x = 1$.
3. Factor: $\cos x(1 - \cos x) = 0$.
4. Solve $\cos x = 0$ or $\cos x = 1$ in the given interval.

Answer: $x = 0, \pi/2, 3\pi/2$

PRACTICE 6 problems

1 Rewrite $\sin^2 x + 2\cos^2 x$ using only cos x.

ANSWER _____

2 Rewrite $3\sin^2 x - \cos^2 x$ using only sin x.

ANSWER _____

3 Solve $\sin^2 x = \cos^2 x$ for $0 \le x < 2\pi$.

ANSWER _____

4 Solve $2\sin^2 x + \sin x = 1$ for $0 \le x < 2\pi$.

ANSWER _____

5 Solve $\cos^2 x + 3\sin x = 1$ for $0 \le x < 2\pi$.

ANSWER _____

6 Solve $4\sin^2 x + 2\cos x = 4$ for $0 \le x < 2\pi$.

ANSWER _____

APPLY IT Show your work

1 A phone animation repeats once as its phase x goes from 0 to 2π radians. Its color setting is $B(x) = 2\sin^2 x + \cos x + 2$. At which phases is $B(x) = 3$?

WORK SPACE

ANSWER _____

2 A music producer sets a looping sound effect to $S(x) = 2\cos^2 x + \sin x + 2$, where x is the phase in radians. During one loop, $0 \le x < 2\pi$, at which phases is $S(x) = 4$?

WORK SPACE

ANSWER _____

CHALLENGE One step up

Solve $\sin^4 x - \cos^4 x = \cos x$ for $0 \le x < 2\pi$. Explain how an identity helps you rewrite the left side.

WORK SPACE AND ANSWER

ANSWER KEY

Identity Detective

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Rewrite $\sin^2 x + 2\cos^2 x$ using only cos x.	**$1 + \cos^2 x$**
2	Rewrite $3\sin^2 x - \cos^2 x$ using only sin x.	**$4\sin^2 x - 1$**
3	Solve $\sin^2 x = \cos^2 x$ for $0 \le x < 2\pi$.	**$x = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$**
4	Solve $2\sin^2 x + \sin x = 1$ for $0 \le x < 2\pi$.	**$x = \pi/6, 5\pi/6, 3\pi/2$**
5	Solve $\cos^2 x + 3\sin x = 1$ for $0 \le x < 2\pi$.	**$x = 0, \pi$**
6	Solve $4\sin^2 x + 2\cos x = 4$ for $0 \le x < 2\pi$.	**$x = \pi/3, \pi/2, 3\pi/2, 5\pi/3$**

PART 2 · APPLY IT

1. **$x = 0, 2\pi/3, 4\pi/3$** Substitute $\sin^2 x = 1 - \cos^2 x$ and factor to get $(2\cos x + 1)(\cos x - 1) = 0$. Thus $\cos x = -1/2$ or $\cos x = 1$.
2. **$x = 0, \pi/6, 5\pi/6, \pi$** Substitute $\cos^2 x = 1 - \sin^2 x$ and factor to get $\sin x(1 - 2\sin x) = 0$. Thus $\sin x = 0$ or $\sin x = 1/2$.

CHALLENGE

Factor the left side as $(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)$. The second factor is 1, so the equation becomes $1 - 2\cos^2 x = \cos x$. Factoring gives $(2\cos x - 1)(\cos x + 1) = 0$, so $x = \pi/3, \pi, 5\pi/3$.

Accept solutions in any order. For the rewrite problems, accept algebraically equivalent expressions that use only the requested trig function.