LESSON MAKER PRO

Volume by Slices

Use cross sections to explain and calculate volume.

HIGH SCHOOL · G.GMD.A.1

G.GMD.A.1 "Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments."

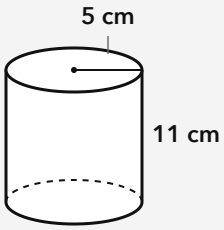
NAME _____ DATE _____

THE BIG IDEA

Stacking equal circular slices gives a cylinder a volume of $\pi r^2 h$. In a cone or pyramid, slices get smaller toward the tip; adding very thin slices gives one-third the volume of a matching cylinder or prism. Cavalieri's principle says two solids have equal volume if their cross sections have equal areas at every matching height.

WORKED EXAMPLE

Explain why you can multiply the base area by the height to find the volume of the solid shown. Then find its volume.

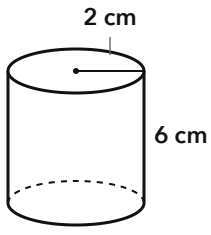


1. Every horizontal slice has the same area as the circular base.
2. Base area = $\pi(5)^2 = 25\pi$ cm^2.
3. Volume = $25\pi \times 11 = 275\pi$ cm^3.

Answer: 275π cm^3

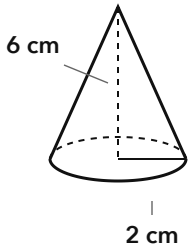
PRACTICE 6 problems

1 Find the volume of the solid shown by multiplying its base area by its height.



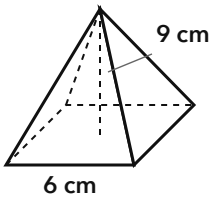
ANSWER _____

2 The cone shown and a cylinder have the same base and height. Find the cone's volume.



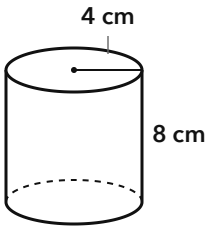
ANSWER _____

3 The pyramid shown has a square base. Find its volume and state how it compares with a prism of the same base and height.



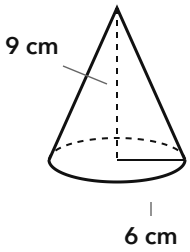
ANSWER _____

4 A prism has a base area of 16π cm^2 and the same height as the cylinder shown. Use Cavalieri's principle to compare their volumes and find each volume.



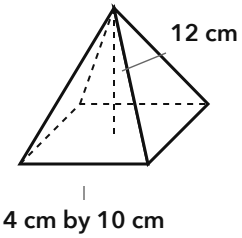
ANSWER _____

5 Find the volume of the cone shown. Explain why you use a factor of 1/3.



ANSWER _____

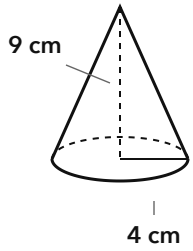
6 Use the dimensions shown to find the volume of this rectangular-based pyramid.



ANSWER _____

APPLY IT Show your work

1 At a concert, a cone-shaped cup has the dimensions shown. A cylindrical cup has the same radius and height. How much more liquid can the cylindrical cup hold?



ANSWER _____

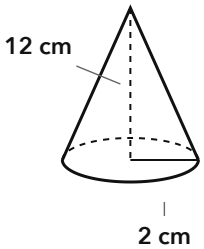
2 A stage crew fills a square-pyramid prop with sand. Its base has a side length of 12 in, and its vertical height is 6 in. Each bag contains 96 in^3 of sand. How many bags are needed to fill the prop?

WORK SPACE

ANSWER _____

CHALLENGE One step up

The cone shown is cone A. Cone B has the same height but three times the radius. How many times greater is cone B's volume? Explain using horizontal cross sections.



WORK SPACE AND ANSWER

ANSWER KEY

Volume by Slices

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the volume of the solid shown by multiplying its base area by its height.	**24π cm^3**
2	The cone shown and a cylinder have the same base and height. Find the cone's volume.	**8π cm^3**
3	The pyramid shown has a square base. Find its volume and state how it compares with a prism of the same base and height.	**108 cm^3; one-third the volume of the matching prism.**
4	A prism has a base area of 16π cm^2 and the same height as the cylinder shown. Use Cavalieri's principle to compare their volumes and find each volume.	**Their volumes are equal; each is 128π cm^3.**
5	Find the volume of the cone shown. Explain why you use a factor of 1/3.	**108π cm^3; its volume is one-third that of a cylinder with the same base and height.**
6	Use the dimensions shown to find the volume of this rectangular-based pyramid.	**160 cm^3**

PART 2 · APPLY IT

1. **96π cm^3** The cylinder holds 144π cm^3, and the cone holds 48π cm^3. The difference is 96π cm^3.
2. **3 bags** The prop holds (1/3)(12 × 12)(6) = 288 in^3. Divide by 96 in^3 per bag.

CHALLENGE

9 times greater. At every matching height, cone B's cross-section radius is three times cone A's, so its cross-section area is nine times as large. Its volume is therefore nine times as large.

Accept equivalent exact expressions with π. For explanations, accept thin-slice reasoning or a comparison with a matching cylinder or prism.