LESSON MAKER PRO

Coordinate Switch

Convert equations between polar and rectangular forms.

HIGH SCHOOL · G.GPE.D.11

G.GPE.D.11 "Convert equations between polar and rectangular forms."

NAME _____ DATE _____

THREE USEFUL IDENTITIES

Use $x = r\cos\theta$, $y = r\sin\theta$, and $r^2 = x^2 + y^2$ to switch between forms. If you multiply or divide by r, check whether the origin was added or lost.

WORKED EXAMPLE

Convert $r = 7\sin\theta$ to rectangular form.

1. Multiply by r: $r^2 = 7r\sin\theta$.
2. Substitute $r^2 = x^2 + y^2$ and $r\sin\theta = y$.
3. Get $x^2 + y^2 = 7y$.
4. Check the origin: $r = 0$ and $\theta = 0$ satisfy the original equation.

Answer: $x^2 + y^2 = 7y$

PRACTICE 6 problems

1 Convert $r = 3\cos\theta$ to rectangular form.

ANSWER _____

2 Convert $r = 5\sin\theta$ to rectangular form.

ANSWER _____

3 Convert $r\sin\theta = -4$ to rectangular form.

ANSWER _____

4 Convert $r^2 = 36$ to rectangular form.

ANSWER _____

5 Convert $x^2 + y^2 = 10x$ to polar form.

ANSWER _____

6 Convert $x = -6$ to polar form.

ANSWER _____

APPLY IT Show your work

1 At an outdoor concert, a phone connects to a speaker along a boundary 9 meters from the speaker. The speaker is at the origin. Write the boundary equation in polar form, then convert it to rectangular form.

WORK SPACE

ANSWER _____

2 In a video game, the boundary of a safe zone is $x^2 + y^2 = 12y$. Convert the boundary equation to polar form.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Convert $r = 4/(1 + \cos \theta)$ to rectangular form. Show why squaring your equation does not add any points.

WORK SPACE AND ANSWER

Coordinate Switch

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Convert $r = 3 \cos \theta$ to rectangular form.	**$x^2 + y^2 = 3x$**
2	Convert $r = 5 \sin \theta$ to rectangular form.	**$x^2 + y^2 = 5y$**
3	Convert $r \sin \theta = -4$ to rectangular form.	**$y = -4$**
4	Convert $r^2 = 36$ to rectangular form.	**$x^2 + y^2 = 36$**
5	Convert $x^2 + y^2 = 10x$ to polar form.	**$r = 10 \cos \theta$**
6	Convert $x = -6$ to polar form.	**$r \cos \theta = -6$**

PART 2 · APPLY IT

1. **$r = 9$; $x^2 + y^2 = 81$** A boundary 9 meters from the origin has $r = 9$. Square both sides and substitute $x^2 + y^2$ for r^2.
2. **$r = 12 \sin \theta$** Substitute r^2 for $x^2 + y^2$ and $r \sin \theta$ for y, then divide by r when $r \neq 0$. The polar equation also includes the origin when $\theta = 0$.

CHALLENGE

Multiply to get $r + r \cos \theta = 4$, so $\sqrt{(x^2 + y^2)} = 4 - x$. Squaring and simplifying gives $y^2 = 16 - 8x$. Every point on this parabola has $x \leq 2$, so $4 - x$ is positive. Thus every point also satisfies the equation before squaring.

Accept equivalent equations. In problem 5 and the second word problem, the simplified polar equations include the origin, so dividing by r does not lose that point.