LESSON MAKER PRO

# Polar Path Crossings

*Find every point shared by two polar curves.*

**HIGH SCHOOL · G.GPE.D.13**

**G.GPE.D.13** "Solve systems of polar equations."

NAME _____ DATE _____

## ONE POINT, TWO NAMES

The polar coordinates (r, θ) and (-r, θ + 180°) name the same point. To find every intersection, check where the curves have equal radii at the same angle, then check for opposite-sign radii at angles 180° apart. Check the pole separately, and list each point once using r ≥ 0 and 0° ≤ θ < 360°.

## WORKED EXAMPLE

Find all intersections of r = 5 and r = 10 cos θ.

1. Set the radii equal: 5 = 10 cos θ, so cos θ = 1/2.
2. At θ = 60° and 300°, this gives (5, 60°) and (5, 300°).
3. The second curve has r = -5 at θ = 120° and 240°. These name the same two points, so they add no intersections.
4. The curve r = 5 does not pass through the pole.

**Answer: (5, 60°) and (5, 300°)**

## PRACTICE 6 problems

**1** Find all intersections of r = 1 and r = 2 sin θ.

**ANSWER** _____

**2** Find all intersections of r = 4 and r = 8 sin θ.

**ANSWER** _____

**3** Find all intersections of r = 2 and r = 2 cos θ.

**ANSWER** _____

**4** Find all intersections of r = 1 and r = 1 + cos θ.

**ANSWER** _____

**5** Find all intersections of r = 4 cos θ and r = 4 sin θ. Include the pole if it is shared.

**ANSWER** _____

**6** Find all intersections of r = 1 and r = cos(2θ).

**ANSWER** _____

## APPLY IT Show your work

**1** At a concert, two projected light edges follow $r = 2$ and $r = 2 + 2\cos\theta$, with distances measured in meters from the stage center. Find the polar coordinates of every point where the edges cross.

WORK SPACE

**ANSWER** _____

**2** Two drones fly looping routes described by $r = 8\cos\theta$ and $r = 8\sin\theta$, with distances measured in meters from their launch point. Where do their routes cross? Give the nonzero point in polar coordinates.

WORK SPACE

**ANSWER** _____

## CHALLENGE One step up

Find all intersections of $r = 2$ and $r = 4\cos(2\theta)$. Explain why setting the radii equal does not find them all.

WORK SPACE AND ANSWER

**ANSWER KEY**

# Polar Path Crossings

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## PART 1 · PRACTICE

| # | PROBLEM | ANSWER |
|---|---|---|
| 1 | Find all intersections of $r = 1$ and $r = 2 \sin \theta$. | **(1, 30°) and (1, 150°)** |
| 2 | Find all intersections of $r = 4$ and $r = 8 \sin \theta$. | **(4, 30°) and (4, 150°)** |
| 3 | Find all intersections of $r = 2$ and $r = 2 \cos \theta$. | **(2, 0°)** |
| 4 | Find all intersections of $r = 1$ and $r = 1 + \cos \theta$. | **(1, 90°) and (1, 270°)** |
| 5 | Find all intersections of $r = 4 \cos \theta$ and $r = 4 \sin \theta$. Include the pole if it is shared. | **The pole and (2√2, 45°)** |
| 6 | Find all intersections of $r = 1$ and $r = \cos(2\theta)$. | **(1, 0°), (1, 90°), (1, 180°), and (1, 270°)** |

## PART 2 · APPLY IT

1. **(2, 90°) and (2, 270°)** Set $2 = 2 + 2 \cos \theta$, giving $\cos \theta = 0$. The second radius is never negative, so there are no additional intersections.
2. **The launch point, or pole, and (4√2, 45°)** Both routes pass through the pole. Equal radii give $\theta = 45°$ or $225°$ and a nonzero distance of $4\sqrt{2}$ meters; those two polar representations name the same point.

## CHALLENGE

**(2, 30°), (2, 60°), (2, 120°), (2, 150°), (2, 210°), (2, 240°), (2, 300°), and (2, 330°). Setting the radii equal gives only the points at 30°, 150°, 210°, and 330°. A radius of -2 on the second curve gives four more points because a negative radius names a point in the opposite direction.**

Accept the intersections in any order and equivalent exact forms of $2\sqrt{2}$ and $4\sqrt{2}$. Count the pole once, regardless of the angle used to represent it.