LESSON MAKER PRO

Fractions in Pieces

Break rational functions into simpler terms.

HIGH SCHOOL · A3.A.14

A3.A.14 "Decompose a rational function into partial fractions."

NAME _____ DATE _____

SET UP THE PIECES

Write one term for each distinct linear factor in the denominator. For a repeated factor, include a term for each power. If a quadratic factor cannot be factored further, use a linear numerator over it. Multiply by the full denominator, then solve for the unknown coefficients.

WORKED EXAMPLE

Decompose (18x+2)/((x-7)(x+9)) into partial fractions.

1. Write (18x+2)/((x-7)(x+9)) = A/(x-7) + B/(x+9).
2. Multiply by (x-7)(x+9): 18x+2 = A(x+9) + B(x-7).
3. Set x=7: 128=16A, so A=8. Set x=-9: -160=-16B, so B=10.

Answer: 8/(x-7) + 10/(x+9)

PRACTICE 6 problems

1 Decompose (3x+4)/((x+1)(x+2)) into partial fractions.

ANSWER _____

2 Decompose (5x-2)/((x-1)(x+2)) into partial fractions.

ANSWER _____

3 Decompose (2x+14)/((x+3)(x+5)) into partial fractions.

ANSWER _____

4 Decompose (5x-8)/(x(x-4)) into partial fractions.

ANSWER _____

5 Decompose $(2x+3)/(x-1)^2$ into partial fractions.

ANSWER _____

6 Decompose $(5x^2+5x+5)/((x+2)(x^2+1))$ into partial fractions.

ANSWER _____

APPLY IT Show your work

1 An esports club models a member's share of two online fees by C(x)=(7x+16)/((x+1)(x+4)) dollars. The first fee is shared by x+1 members, and the second is shared by x+4 members. Decompose C(x), then find the total amount of each fee.

WORK SPACE

ANSWER _____

2 An app downloads two batches of files one after the other. Their speeds are x+2 and x+3 files per second, respectively. The total time is T(x)=(5x+13)/((x+2)(x+3)) seconds. Decompose T(x), then find how many files are in each batch.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Decompose $(3x^2+3x+3)/((x-1)^2(x+2))$ into partial fractions. Explain why your setup needs three terms.

WORK SPACE AND ANSWER

ANSWER KEY # Fractions in Pieces

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Decompose (3x+4)/((x+1)(x+2)) into partial fractions.	**1/(x+1) + 2/(x+2)**
2	Decompose (5x-2)/((x-1)(x+2)) into partial fractions.	**1/(x-1) + 4/(x+2)**
3	Decompose (2x+14)/((x+3)(x+5)) into partial fractions.	**4/(x+3) - 2/(x+5)**
4	Decompose (5x-8)/(x(x-4)) into partial fractions.	**2/x + 3/(x-4)**
5	Decompose (2x+3)/$(x-1)^2$ into partial fractions.	**2/(x-1) + 5/$(x-1)^2$**
6	Decompose ($5x^2$+5x+5)/((x+2)(x^2+1)) into partial fractions.	**3/(x+2) + (2x+1)/(x^2+1)**

PART 2 · APPLY IT

1. **C(x)=3/(x+1) + 4/(x+4). The first fee is $3, and the second fee is $4.** Set C(x)=A/(x+1)+B/(x+4). Then 7x+16=A(x+4)+B(x+1), giving A=3 and B=4.
2. **T(x)=3/(x+2) + 2/(x+3). The first batch has 3 files, and the second has 2 files.** Set T(x)=A/(x+2)+B/(x+3). Matching coefficients in 5x+13=A(x+3)+B(x+2) gives A=3 and B=2.

CHALLENGE

2/(x-1) + 3/$(x-1)^2$ + 1/(x+2). The repeated factor x-1 needs a term for each of its two powers, and x+2 needs one more term.

Accept equivalent partial-fraction terms in any order. For the challenge, the explanation must account for both powers of x-1 and the factor x+2.