LESSON MAKER PRO

Missing Points

Find holes, asymptotes, and breaks in continuity.

HIGH SCHOOL · A3.A.15

A3.A.15 "Determine asymptotes and holes of rational functions, explain how each was found, and relate these behaviors to continuity."

NAME _____ DATE _____

FACTOR FIRST

Cancel common factors, but keep their zeros excluded from the original function. A canceled factor gives a hole if the reduced denominator is nonzero there; a zero of the reduced denominator gives a vertical asymptote. Compare degrees and leading coefficients for the horizontal asymptote. A function is not continuous at a hole or a vertical asymptote.

WORKED EXAMPLE

Find the hole and asymptotes of f(x)=(x-11)(x+13)/((x-11)(x-17)). Where is f not continuous?

VALUES NEAR X = 11

x	f(x)
10	-23/7
12	-5

1. Cancel x-11 to get (x+13)/(x-17). Keep x=11 excluded.
2. At x=11, the reduced expression gives 24/-6=-4.
3. The remaining denominator is zero at x=17.
4. The degrees match, and the leading-coefficient ratio is 1.

Answer: Hole: (11,-4). Vertical asymptote: x=17. Horizontal asymptote: y=1. The function is not continuous at x=11 or x=17.

PRACTICE 4 problems

1 Find the hole and vertical asymptote of f(x)=(x+3)(x-4)/((x+3)(x-1)).

ANSWER _____

2 Find both asymptotes of f(x)=(x-5)/(x+2).

ANSWER _____

3 For f(x)=(x-6)(x+7)/((x-6)(x+8)), name each x-value where f is not continuous. Classify each break.

ANSWER _____

4 Find the horizontal asymptote of f(x)=(x+9)/(x^2-25).

ANSWER _____

APPLY IT Show your work

1 A gaming app models server wait time in minutes by $W(x)=(x-3)(x+5)/((x-3)(x-1))$, where $x>1$ is minutes since a reset. Find the hole and vertical asymptote. Is W continuous at $x=3$?

WORK SPACE

ANSWER _____

2 An app models upload time in seconds by $T(x)=(2x+7)/(x-6)$, where $x>6$ is available bandwidth. Find both asymptotes. What happens to T(x) as x approaches 6 from above?

WORK SPACE

ANSWER _____

CHALLENGE One step up

Factor $f(x)=(x^2-2x-8)/(x^2-7x+12)$. Find its hole and both asymptotes. Which break in continuity could be fixed by defining one function value?

WORK SPACE AND ANSWER

ANSWER KEY

Missing Points

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the hole and vertical asymptote of f(x)=(x+3)(x-4)/((x+3)(x-1)).	**Hole: (-3,7/4). Vertical asymptote: x=1.**
2	Find both asymptotes of f(x)=(x-5)/(x+2).	**Vertical asymptote: x=-2. Horizontal asymptote: y=1.**
3	For f(x)=(x-6)(x+7)/((x-6)(x+8)), name each x-value where f is not continuous. Classify each break.	**x=6 is a hole; x=-8 is a vertical asymptote.**
4	Find the horizontal asymptote of $f(x)=(x+9)/(x^2-25)$.	**y=0**

PART 2 · APPLY IT

1. **Hole: (3,4). Vertical asymptote: x=1. W is not continuous at x=3.** Cancel x-3, then substitute x=3 into (x+5)/(x-1). The remaining denominator is zero at x=1, while the excluded value x=3 makes a hole.
2. **Vertical asymptote: x=6. Horizontal asymptote: y=2. T(x) increases without bound as x approaches 6 from above.** The denominator approaches zero from above while the numerator approaches 19. The leading-coefficient ratio is 2.

CHALLENGE

f(x)=(x-4)(x+2)/((x-4)(x-3)). Hole: (4,6). Vertical asymptote: x=3. Horizontal asymptote: y=1. Defining f(4)=6 would fix the break at x=4.

Accept equivalent factored expressions. A hole is a point, while vertical and horizontal asymptotes are lines.