LESSON MAKER PRO

Function Switchboard

Compose functions, reverse them, and find the identity.

HIGH SCHOOL · A3.A.18

A3.A.18 "Find the composite of two given functions and find the inverse of a given function. Extend this concept to discuss the identity function f(x) = x."

NAME _____ DATE _____

THE BIG IDEA

To find f(g(x)), put the output of g into f, then simplify. To find an inverse, write y = f(x), switch x and y, and solve for y. A function and its inverse undo each other, so their composition is the identity function, which gives back x.

WORKED EXAMPLE

Find the inverse of f(x) = 7x + 19. Then check $f(f^{-1}(x))$.

1. Write y = 7x + 19.
2. Switch x and y: x = 7y + 19.
3. Solve for y: y = (x - 19)/7.
4. Check: $f(f^{-1}(x)) = 7((x - 19)/7) + 19 = x$.

Answer: $f^{-1}(x) = (x - 19)/7$, and $f(f^{-1}(x)) = x$.

PRACTICE 6 problems

1 Let f(x) = 2x + 5 and g(x) = x - 4. Find f(g(x)).

ANSWER _____

2 Let $f(x) = x^2 + 6$ and g(x) = x + 2. Find f(g(x)).

ANSWER _____

3 Let f(x) = 5x - 9 and g(x) = 2x + 4. Find g(f(x)).

ANSWER _____

4 Find the inverse of f(x) = 4x - 6.

ANSWER _____

5 Find the inverse of f(x) = (x + 10)/5.

ANSWER _____

6 Let f(x) = 2x - 5 and g(x) = (x + 5)/2. Find both f(g(x)) and g(f(x)).

ANSWER _____

APPLY IT Show your work

1 A rideshare app charges $4 to start a trip and $2 per mile. A rider then adds a $6 tip. Let $F(x) = 2x + 4$ be the fare for x miles, and let $T(x) = x + 6$ add the tip to a dollar amount. Find $T(F(x))$, then find the total for a 5-mile trip.

WORK SPACE

ANSWER _____

2 A student earns $9 per hour tutoring and gets a one-time $15 bonus. Let $E(x) = 9x + 15$ be the total earnings for x hours. Find $E^{-1}(x)$, then use it to find how many hours the student tutored if the total was $51.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Let $f(x) = (2x - 4)/(x + 5)$, where x is not -5. Find $f^{-1}(x)$ and state its excluded input. Show that $f(f^{-1}(x)) = x$ for every allowed input.

WORK SPACE AND ANSWER

Function Switchboard

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Let f(x) = 2x + 5 and g(x) = x - 4. Find f(g(x)).	**2x - 3**
2	Let f(x) = x^2 + 6 and g(x) = x + 2. Find f(g(x)).	**x^2 + 4x + 10**
3	Let f(x) = 5x - 9 and g(x) = 2x + 4. Find g(f(x)).	**10x - 14**
4	Find the inverse of f(x) = 4x - 6.	**$f^{-1}(x) = (x + 6)/4$**
5	Find the inverse of f(x) = (x + 10)/5.	**$f^{-1}(x) = 5x - 10$**
6	Let f(x) = 2x - 5 and g(x) = (x + 5)/2. Find both f(g(x)) and g(f(x)).	**f(g(x)) = x; g(f(x)) = x**

PART 2 · APPLY IT

1. **T(F(x)) = 2x + 10; the 5-mile total is $20.** Add the $6 tip to the fare to get T(F(x)) = (2x + 4) + 6. Substitute 5 for x.
2. **$E^{-1}(x) = (x - 15)/9$; the student tutored for 4 hours.** Subtract the bonus and divide by the hourly rate to reverse E. Then $E^{-1}(51) = (51 - 15)/9 = 4$.

CHALLENGE

$f^{-1}(x) = (5x + 4)/(2 - x)$, with x not 2. Substituting gives $f(f^{-1}(x)) = [2(5x + 4) - 4(2 - x)]/[(5x + 4) + 5(2 - x)] = 14x/14 = x$.

Accept equivalent simplified expressions. In the challenge, students must exclude 2 from the inverse's inputs and show that the composition simplifies to x.