LESSON MAKER PRO

Fraction Inside Fraction

Simplify complex fractions and difference quotients

HIGH SCHOOL · A3.A.19

A3.A.19 "Simplify complex algebraic fractions (with/without variable expressions and integer exponents) to include expressing (f(x + h) – f(x))/h as single simplified fraction when f(x) = 1/(1–x) for example."

NAME _____ **DATE** _____

CLEAR AND CANCEL

To simplify a fraction that contains smaller fractions, first combine the terms in its numerator or denominator. For a difference quotient, combine the function values before canceling h. You can cancel h only when it is a factor of the entire numerator and h is not zero.

WORKED EXAMPLE

Simplify (1/(8-x-h) - 1/(8-x))/h. Assume all original denominators are nonzero.

1. Use a common denominator: ((8-x) - (8-x-h))/[h(8-x-h)(8-x)].
2. Simplify the numerator: h/[h(8-x-h)(8-x)].
3. Cancel h: 1/[(8-x-h)(8-x)].

Answer: 1/[(8-x-h)(8-x)]

PRACTICE 6 problems

1 Simplify (1/x + 1/3)/(2/x). Assume all original denominators are nonzero.

ANSWER _____

2 Simplify $(x^{-1} - 1/5)/(x^{-1} + 1/5)$. Assume all original denominators are nonzero.

ANSWER _____

3 Simplify (2/x - 3/(x+1))/(1/x). Assume all original denominators are nonzero.

ANSWER _____

4 Simplify (1/(1-x-h) - 1/(1-x))/h. Assume all original denominators are nonzero.

ANSWER _____

5 Simplify (1/(x+h+2) - 1/(x+2))/h. Assume all original denominators are nonzero.

ANSWER _____

6 Let f(x) = 3/(x-4). Simplify (f(x+h) - f(x))/h. Assume all original denominators are nonzero.

ANSWER _____

APPLY IT Show your work

1 A 6 MB file takes T(x) = 6/x hours to upload at a speed of x MB per hour. For x > 0 and h > 0, simplify (T(x+h) - T(x))/h. What does its sign tell you about upload time as speed increases?

WORK SPACE

ANSWER _____

2 One group splits a $15 game subscription among x members. Another group splits a $20 subscription among x+1 members. For x > 0, simplify the ratio of the first group's cost per member to the second group's cost per member.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Let f(x) = (x+1)/(x-2). Simplify (f(x+h) - f(x))/h, assuming all original denominators are nonzero. Explain why you can cancel h.

WORK SPACE AND ANSWER

Fraction Inside Fraction

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Simplify (1/x + 1/3)/(2/x). Assume all original denominators are nonzero.	**(x+3)/6**
2	Simplify (x^{-1} - 1/5)/(x^{-1} + 1/5). Assume all original denominators are nonzero.	**(5-x)/(5+x)**
3	Simplify (2/x - 3/(x+1))/(1/x). Assume all original denominators are nonzero.	**(2-x)/(x+1)**
4	Simplify (1/(1-x-h) - 1/(1-x))/h. Assume all original denominators are nonzero.	**1/[(1-x-h)(1-x)]**
5	Simplify (1/(x+h+2) - 1/(x+2))/h. Assume all original denominators are nonzero.	**-1/[(x+h+2)(x+2)]**
6	Let f(x) = 3/(x-4). Simplify (f(x+h) - f(x))/h. Assume all original denominators are nonzero.	**-3/[(x+h-4)(x-4)]**

PART 2 · APPLY IT

1. **-6/[x(x+h)]; the negative sign means upload time decreases as speed increases.** Combine 6/(x+h) - 6/x to get -6h/[x(x+h)], then divide by h. Both factors in the denominator are positive, so the result is negative.
2. **3(x+1)/(4x)** The ratio is (15/x)/(20/(x+1)). Multiply by the reciprocal and reduce 15/20 to 3/4.

CHALLENGE

-3/[(x+h-2)(x-2)]. Combining the function values gives -3h/[h(x+h-2)(x-2)], so h cancels because it is a factor and h is not zero.

Accept equivalent simplified forms. Original denominator restrictions still apply after factors cancel.