LESSON MAKER PRO

Zero Detective

Use a known zero to uncover the rest.

HIGH SCHOOL · A3.A.21

A3.A.21 "Find the zeros of polynomial functions by synthetic division and the Factor Theorem."

NAME _____ **DATE** _____

DIVIDE TO FIND

The Factor Theorem says that r is a zero of a polynomial when dividing by x - r leaves a remainder of 0. Use synthetic division to find that remainder and a simpler polynomial, then factor the simpler polynomial to find the other zeros.

WORKED EXAMPLE

Find all zeros of $f(x) = x^3 - 12x^2 - x + 252$, given that -4 is a zero.

1. Write the coefficients in order: 1, -12, -1, 252.
2. Divide synthetically by -4. The products are -4, 64, -252, and the bottom row is 1, -16, 63, 0.
3. The remainder is 0, so x + 4 is a factor. The quotient is $x^2 - 16x + 63$.
4. Factor the quotient: $x^2 - 16x + 63 = (x - 7)(x - 9)$.

Answer: The zeros are -4, 7, and 9.

PRACTICE 6 problems

1 Given that -2 is a zero of $f(x) = x^3 - 2x^2 - 5x + 6$, use synthetic division to find all zeros.

ANSWER _____

2 Use the Factor Theorem to check whether 5 is a zero of $f(x) = x^3 - 4x^2 - 5x$. Then use synthetic division to find all zeros.

ANSWER _____

3 Given that -5 is a zero of $f(x) = x^3 + 4x^2 - 7x - 10$, use synthetic division to find all zeros.

ANSWER _____

4 Given that 3 is a zero of $f(x) = x^3 - 4x^2 - 3x + 18$, use synthetic division to find all zeros. List a repeated zero twice.

ANSWER _____

5 Given that -1 is a zero of $f(x) = x^3 - 2x^2 - x + 2$, use synthetic division to find all zeros.

ANSWER _____

6 Given that -5 is a zero of $f(x) = x^4 - 19x^2 + 30x$, use synthetic division to find all zeros. Include 0 for any missing powers when you set up the division.

ANSWER _____

APPLY IT Show your work

1 A student selling custom stickers models profit with $P(x) = x^3 - 15x^2 + 66x - 80$, where x is the number of dozens sold and P(x) is in dollars. The student knows that selling 2 dozen gives a profit of $0. Use synthetic division to find every positive whole-number value of x at which the model predicts a profit of $0. State your answers in dozens.

WORK SPACE

ANSWER _____

2 A video editor models the difference between two clips' view counts, in thousands of views, as $D(x) = x^3 - 9x^2 + 26x - 24$, where x is days after posting. On day 2, the modeled view counts are equal. Use synthetic division to find every day from 0 through 5 when the model predicts equal view counts.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Find k so that $x + 1$ is a factor of $f(x) = x^4 + kx^3 - 3x^2 + 7x + 6$. Then use synthetic division to find all zeros, listing any repeated zero twice. Explain how you found k.

WORK SPACE AND ANSWER

ANSWER KEY # Zero Detective

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Given that -2 is a zero of $f(x) = x^3 - 2x^2 - 5x + 6$, use synthetic division to find all zeros.	**x = -2, 1, 3**
2	Use the Factor Theorem to check whether 5 is a zero of $f(x) = x^3 - 4x^2 - 5x$. Then use synthetic division to find all zeros.	**f(5) = 0, so 5 is a zero. All zeros are x = -1, 0, 5.**
3	Given that -5 is a zero of $f(x) = x^3 + 4x^2 - 7x - 10$, use synthetic division to find all zeros.	**x = -5, -1, 2**
4	Given that 3 is a zero of $f(x) = x^3 - 4x^2 - 3x + 18$, use synthetic division to find all zeros. List a repeated zero twice.	**x = -2, 3, 3**
5	Given that -1 is a zero of $f(x) = x^3 - 2x^2 - x + 2$, use synthetic division to find all zeros.	**x = -1, 1, 2**
6	Given that -5 is a zero of $f(x) = x^4 - 19x^2 + 30x$, use synthetic division to find all zeros. Include 0 for any missing powers when you set up the division.	**x = -5, 0, 2, 3**

PART 2 · APPLY IT

1. **2 dozen, 5 dozen, and 8 dozen** Dividing by 2 gives $x^2 - 13x + 40$ with remainder 0. The quotient factors as $(x - 5)(x - 8)$.
2. **Days 2, 3, and 4** Dividing by 2 gives $x^2 - 7x + 12$ with remainder 0. The quotient factors as $(x - 3)(x - 4)$.

CHALLENGE

By the Factor Theorem, f(-1) = -3 - k must equal 0, so k = -3. Synthetic division by -1 gives $x^3 - 4x^2 + x + 6 = (x + 1)(x - 2)(x - 3)$. The zeros are x = -1, -1, 2, 3.

Accept zeros in any order. For the challenge, accept a statement that the remainder is 0 when k = -3 as the explanation.