LESSON MAKER PRO

Which Way Next?

Predict where a polynomial heads at both ends.

HIGH SCHOOL · A3.A.9

A3.A.9 "Determine the end behavior of polynomial functions."

NAME _____ DATE _____

FOLLOW THE LEADING TERM

For very large positive or negative values of x, a polynomial's leading term determines its end behavior. An even power gives the same direction on both ends, while an odd power gives opposite directions. A positive leading coefficient points the right end up, and a negative one points it down.

WORKED EXAMPLE

Determine the end behavior of $f(x) = -7x^9 + 11x^7 - 13$.

1. The leading term is $-7x^9$.
2. Its power is odd, so the ends point in opposite directions.
3. Its coefficient is negative, so the right end points down and the left end points up.

Answer: As $x \to -\infty$, $f(x) \to \infty$. As $x \to \infty$, $f(x) \to -\infty$.

PRACTICE 6 problems

1 State the end behavior of $f(x) = 3x^5 - 2x + 8$ as $x \to -\infty$ and as $x \to \infty$.

ANSWER _____

2 State the end behavior of $g(x) = -5x^6 + 10x^3 - 1$ as $x \to -\infty$ and as $x \to \infty$.

ANSWER _____

3 State the end behavior of $h(x) = -2x^3 + 8x^2 + 6$ as $x \to -\infty$ and as $x \to \infty$.

ANSWER _____

4 State the end behavior of $p(x) = 6 - 8x^6 + 12x^8$ as $x \to -\infty$ and as $x \to \infty$.

ANSWER _____

5 State the end behavior of $q(x) = (x - 2)(x + 3)(x - 5)$ as $x \to -\infty$ and as $x \to \infty$.

ANSWER _____

6 State the end behavior of $r(x) = -3x^3(x - 6)(x + 2)$ as $x \to -\infty$ and as $x \to \infty$.

ANSWER _____

APPLY IT Show your work

1 A skate park designer models a ramp's height above the ground, in meters, with $H(x) = -2x^4 + 3x^2 + 6$. Here x is the horizontal distance in meters from the ramp's center, with negative x to the left. Determine the model's end behavior in both directions. Why can't this model describe a ramp for every value of x?

WORK SPACE

ANSWER _____

2 An esports analyst models Team A's score minus Team B's score as $D(x) = 2x^5 - 3x^2 + 10$, where x is Team A's skill rating minus Team B's. Determine the model's end behavior as $x \to -\infty$ and as $x \to \infty$. Which team does the model predict will lead when x is very negative? When x is very positive?

WORK SPACE

ANSWER _____

CHALLENGE One step up

Determine the end behavior of $C(x) = (2x^5 - 3x^2 + 8) - (2x^5 + 6x^4 - 5x)$. Explain why looking only at the highest power shown in the original expression would give the wrong answer.

WORK SPACE AND ANSWER

Which Way Next?

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	State the end behavior of $f(x) = 3x^5 - 2x + 8$ as $x \to -\infty$ and as $x \to \infty$.	**As $x \to -\infty$, $f(x) \to -\infty$. As $x \to \infty$, $f(x) \to \infty$.**
2	State the end behavior of $g(x) = -5x^6 + 10x^3 - 1$ as $x \to -\infty$ and as $x \to \infty$.	**As $x \to -\infty$, $g(x) \to -\infty$. As $x \to \infty$, $g(x) \to -\infty$.**
3	State the end behavior of $h(x) = -2x^3 + 8x^2 + 6$ as $x \to -\infty$ and as $x \to \infty$.	**As $x \to -\infty$, $h(x) \to \infty$. As $x \to \infty$, $h(x) \to -\infty$.**
4	State the end behavior of $p(x) = 6 - 8x^6 + 12x^8$ as $x \to -\infty$ and as $x \to \infty$.	**As $x \to -\infty$, $p(x) \to \infty$. As $x \to \infty$, $p(x) \to \infty$.**
5	State the end behavior of $q(x) = (x - 2)(x + 3)(x - 5)$ as $x \to -\infty$ and as $x \to \infty$.	**As $x \to -\infty$, $q(x) \to -\infty$. As $x \to \infty$, $q(x) \to \infty$.**
6	State the end behavior of $r(x) = -3x^3(x - 6)(x + 2)$ as $x \to -\infty$ and as $x \to \infty$.	**As $x \to -\infty$, $r(x) \to \infty$. As $x \to \infty$, $r(x) \to -\infty$.**

PART 2 · APPLY IT

1. **As $x \to -\infty$, $H(x) \to -\infty$, and as $x \to \infty$, $H(x) \to -\infty$. Far enough from the center, the model gives a negative height above the ground.** The leading term, $-2x^4$, is negative for large values of x in either direction. A ramp's height above the ground cannot be negative.
2. **As $x \to -\infty$, $D(x) \to -\infty$, so Team B is predicted to lead. As $x \to \infty$, $D(x) \to \infty$, so Team A is predicted to lead.** The leading term, $2x^5$, is negative when x is very negative and positive when x is very positive. A negative score difference favors Team B, while a positive difference favors Team A.

CHALLENGE

The x^5 terms cancel, leaving $C(x) = -6x^4 - 3x^2 + 5x + 8$. Its leading term is $-6x^4$, so as $x \to -\infty$, $C(x) \to -\infty$, and as $x \to \infty$, $C(x) \to -\infty$. The highest power shown does not determine the end behavior because its terms cancel.

Accept equivalent descriptions such as left end up or down and right end up or down when both directions are clear. For the challenge, students must account for cancellation before identifying the leading term.