LESSON MAKER PRO

Patterns to Formulas

Write rules for terms and sums.

HIGH SCHOOL · A3.N.1

A3.N.1 "Express sequences and series using recursive and explicit formulas."

NAME _____ DATE _____

TWO WAYS TO WRITE A RULE

A recursive formula gives the first term and tells you how to get each term from the one before it. An explicit formula finds any term directly: for an arithmetic sequence, $a_n = a_1 + (n - 1)d$; for a geometric sequence, $a_n = a_1 r^{n-1}$. For a series, S_n is the sum of the first n terms.

WORKED EXAMPLE

An arithmetic sequence begins 17, 26, 35, ... Write a recursive and an explicit formula for a_n.

1. The first term is 17, and the common difference is 26 - 17 = 9.
2. For the recursive rule, start at 17 and add 9 to the previous term.
3. For the explicit rule, substitute $a_1 = 17$ and $d = 9$ into $a_n = a_1 + (n - 1)d$.

Answer: Recursive: $a_1 = 17$; $a_n = a_{n-1} + 9$ for $n >= 2$. Explicit: $a_n = 17 + 9(n - 1)$.

PRACTICE 6 problems

1 The sequence is 5, 8, 11, 14, ... Write an explicit formula for a_n.

ANSWER _____

2 The sequence is 30, 24, 18, 12, ... Write a recursive formula for a_n, including the first term.

ANSWER _____

3 A sequence has $a_1 = 3$ and $a_n = a_{n-1} + 5$ for $n >= 2$. Write an explicit formula.

ANSWER _____

4 The sequence is 2, 10, 50, 250, ... Write an explicit formula for a_n.

ANSWER _____

5 The sequence is 64, 32, 16, 8, ... Write a recursive formula for a_n, including the first term.

ANSWER _____

6 A sequence has $a_1 = 3$ and $a_n = 2a_{n-1}$ for $n >= 2$. Write an explicit formula and find a_5.

ANSWER _____

APPLY IT Show your work

1 You set aside $12 in week 1. Each week after that, you set aside $5 more than you set aside the week before. Let a_n be the amount you set aside during week n. Write recursive and explicit formulas, then find the amount for week 8.

WORK SPACE

ANSWER _____

2 A creator's video gets 80 new views on Monday. On each of the next three days, it gets half as many new views as the day before. Let a_n be the new views on day n, with Monday as day 1. Write recursive and explicit formulas for days 1 through 4, then find the new views on day 4.

WORK SPACE

ANSWER _____

CHALLENGE One step up

A concert venue sells 4 tickets in the first minute, 7 in the second, 10 in the third, and continues this pattern. Let S_n be the total tickets sold through minute n. Write an explicit formula and a recursive formula for S_n, then find S_8. Explain how you found the amount added in the recursive formula.

WORK SPACE AND ANSWER

Patterns to Formulas

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	The sequence is 5, 8, 11, 14, … . Write an explicit formula for a_n.	**$a_n = 5 + 3(n - 1)$**
2	The sequence is 30, 24, 18, 12, … . Write a recursive formula for a_n, including the first term.	**$a_1 = 30$; $a_n = a_{n - 1} - 6$ for $n >= 2$**
3	A sequence has $a_1 = 3$ and $a_n = a_{n - 1} + 5$ for $n >= 2$. Write an explicit formula.	**$a_n = 3 + 5(n - 1)$**
4	The sequence is 2, 10, 50, 250, … . Write an explicit formula for a_n.	**$a_n = 2(5)^{n - 1}$**
5	The sequence is 64, 32, 16, 8, … . Write a recursive formula for a_n, including the first term.	**$a_1 = 64$; $a_n = (1/2)a_{n - 1}$ for $n >= 2$**
6	A sequence has $a_1 = 3$ and $a_n = 2a_{n - 1}$ for $n >= 2$. Write an explicit formula and find a_5.	**$a_n = 3(2)^{n - 1}$; $a_5 = 48$**

PART 2 · APPLY IT

1. **Recursive: $a_1 = 12$; $a_n = a_{n - 1} + 5$ for $n >= 2$. Explicit: $a_n = 12 + 5(n - 1)$. Week 8: $47.** The weekly amounts form an arithmetic sequence with first term 12 and common difference 5. Substitute $n = 8$ into the explicit formula to get $12 + 5(7) = 47$.
2. **Recursive: $a_1 = 80$; $a_n = (1/2)a_{n - 1}$ for $2 <= n <= 4$. Explicit: $a_n = 80(1/2)^{n - 1}$ for $1 <= n <= 4$. Day 4: 10 new views.** The daily counts form a geometric sequence with first term 80 and ratio 1/2. On day 4, $80(1/2)^3 = 10$.

CHALLENGE

Explicit: $S_n = n(3n + 5)/2$. Recursive: $S_1 = 4$; $S_n = S_{n - 1} + (3n + 1)$ for $n >= 2$. $S_8 = 116$ tickets. The tickets sold in minute n follow the explicit rule $4 + 3(n - 1) = 3n + 1$, so that amount is added to the previous total.

Accept algebraically equivalent explicit formulas. Recursive formulas must include a starting value and a rule that produces each later term.