LESSON MAKER PRO

# Sum or Spiral

*Find each series limit, or write "diverges" if no finite limit exists.*

**HIGH SCHOOL · A3.N.3**

**A3.N.3** "Calculate limits based on convergent and divergent series."

NAME _____ DATE _____

## GEOMETRIC SERIES LIMITS

In a geometric series, each term is the previous term multiplied by the same number, called the common ratio r. If the first term is a and |r| < 1, the partial sums approach a/(1-r). If |r| is 1 or greater and a is not zero, the partial sums do not approach a finite limit.

## WORKED EXAMPLE

Find the limit of 15 + 5 + 5/3 + ... .

1. The first term is a = 15, and the common ratio is r = 5/15 = 1/3.
2. Since |r| < 1, the limit is a/(1-r).
3. 15/(1-1/3) = 15/(2/3) = 45/2.

**Answer: 45/2**

## PRACTICE 6 problems

**1** Find the limit of 12 + 3 + 3/4 + ... .

**ANSWER** _____

**2** Find the limit of 9 - 3 + 1 - 1/3 + ... .

**ANSWER** _____

**3** Find the limit of 2 + 6 + 18 + ... .

**ANSWER** _____

**4** Find the limit of 6 - 6 + 6 - 6 + ... .

**ANSWER** _____

**5** Find the limit of 14 + 7/2 + 7/8 + ... .

**ANSWER** _____

**6** Find the limit of 3 - 6 + 12 - 24 + ... .

**ANSWER** _____

## APPLY IT Show your work

**1** A phone backup uploads 48 MB in the first minute. It uploads half as much in each minute that follows. If this pattern continues without end, what is the limit of the total data uploaded?

WORK SPACE

**ANSWER** _____

**2** A game awards 11 coins in round 1 and doubles the award in each new round. As the rounds continue without end, does the total number of coins awarded approach a finite limit? Explain.

WORK SPACE

**ANSWER** _____

## CHALLENGE One step up

Find x if the limit of x + x/4 + x/16 + ... is 28. Show how you know.

WORK SPACE AND ANSWER

**ANSWER KEY** # Sum or Spiral



## PART 1 · PRACTICE

| # | PROBLEM | ANSWER |
|---|---|---|
| 1 | Find the limit of 12 + 3 + 3/4 + ... . | **16** |
| 2 | Find the limit of 9 - 3 + 1 - 1/3 + ... . | **27/4** |
| 3 | Find the limit of 2 + 6 + 18 + ... . | **diverges** |
| 4 | Find the limit of 6 - 6 + 6 - 6 + ... . | **diverges** |
| 5 | Find the limit of 14 + 7/2 + 7/8 + ... . | **56/3** |
| 6 | Find the limit of 3 - 6 + 12 - 24 + ... . | **diverges** |

## PART 2 · APPLY IT

1. **96 MB** The amounts form a geometric series with a = 48 and r = 1/2. The limit is 48/(1-1/2) = 96 MB.
2. **No. The total diverges.** The awards form a geometric series with r = 2. Its partial sums keep increasing and do not approach a finite limit.

## CHALLENGE

**x = 21. The common ratio is 1/4, so x/(1-1/4) = 28. Thus 4x/3 = 28 and x = 21.**

Accept equivalent fractions for finite limits. For the game problem, accept an explanation that the coin awards double, so the total keeps growing.