LESSON MAKER PRO

Sums Without End

Find the total when a pattern continues forever.

HIGH SCHOOL · A3.N.4

A3.N.4 "Evaluate and apply infinite geometric series."

NAME _____ DATE _____

WHEN DOES IT ADD UP?

In a geometric series, each term is multiplied by the same number, called the common ratio r. If the first term is a and -1 < r < 1, the infinite series has sum S = a/(1 - r). If r is outside that range, the series does not have a finite sum.

WORKED EXAMPLE

Find the sum of 25 + 10 + 4 + ...

1. Find the common ratio: r = 10/25 = 2/5.
2. Since -1 < 2/5 < 1, use S = a/(1 - r).
3. Substitute: S = 25/(1 - 2/5) = 25/(3/5).
4. Divide: S = 125/3.

Answer: 125/3

PRACTICE 6 problems

1 Find the sum of 12 + 6 + 3 + ...

ANSWER _____

2 Find the sum of 9 - 3 + 1 - 1/3 + ...

ANSWER _____

3 Find the sum of 21 + 7 + 7/3 + ...

ANSWER _____

4 An infinite geometric series has first term 11 and common ratio -1/2. Find its sum.

ANSWER _____

5 Does 5 + 15 + 45 + ... have a finite sum? Give a reason.

ANSWER _____

_____

6 Find the sum of 3/8 + 3/16 + 3/32 + ...

ANSWER _____

APPLY IT Show your work

1 A basketball falls 6 feet to the floor. After each bounce, it rises to 1/3 of the height it just fell. How far does the ball travel in all, including every rise and fall?

WORK SPACE

ANSWER _____

2 A student earns $48 from a video in its first month. A model predicts that each later month will earn half as much as the month before. According to this model, how much will the video earn over all months?

WORK SPACE

ANSWER _____

CHALLENGE One step up

An infinite geometric series starts with 36 and has a sum of 24. Find its common ratio and its next two terms. Explain why the sum exists.

WORK SPACE AND ANSWER

Sums Without End

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the sum of 12 + 6 + 3 + ...	**24**
2	Find the sum of 9 - 3 + 1 - 1/3 + ...	**27/4**
3	Find the sum of 21 + 7 + 7/3 + ...	**63/2**
4	An infinite geometric series has first term 11 and common ratio -1/2. Find its sum.	**22/3**
5	Does 5 + 15 + 45 + ... have a finite sum? Give a reason.	**No. Its common ratio is 3, so the terms do not approach 0.**
6	Find the sum of 3/8 + 3/16 + 3/32 + ...	**3/4**

PART 2 · APPLY IT

1. **12 feet** The first fall is 6 feet. The bounce heights sum to 2/(1 - 1/3) = 3 feet, and each bounce height is traveled twice, so the total is 6 + 2(3) = 12 feet.
2. **$96** The earnings form an infinite geometric series with first term 48 and ratio 1/2. Its sum is 48/(1 - 1/2) = 96 dollars.

CHALLENGE

The common ratio is -1/2, and the next two terms are -18 and 9. Since -1 < -1/2 < 1, the series has a finite sum.

Accept equivalent fractions or decimals for finite sums. For the challenge, students may explain that the terms approach 0 because the absolute value of the ratio is less than 1.