LESSON MAKER PRO

# Count the Possibilities

*Use summation and factorials to count outcomes.*

**HIGH SCHOOL · A3.S.45**

**A3.S.45** "Analyze expressions in summation and factorial notation to solve problems."

NAME _____ DATE _____

### ADD OR COUNT?

The symbol Σ tells you to add an expression's values as its index goes from the lower number to the upper number. A factorial multiplies the positive whole numbers down to 1, and 0! = 1. To count groups of r chosen from n when order does not matter, use n!/(r!(n-r)!).

### WORKED EXAMPLE

Evaluate $\Sigma_{k=10}^{11}\ k!/(k-1)!$.

1. Expand the sum: 10!/9! + 11!/10!.
2. Cancel matching factors: 10 + 11.
3. Add: 21.

**Answer: 21**

## PRACTICE 6 problems

**1** Evaluate $\Sigma_{i=1}^{4}\ (2i-1)$.

**ANSWER** _____

**2** Evaluate 6!/4!.

**ANSWER** _____

**3** Evaluate $\Sigma_{k=1}^{3}\ k!$.

**ANSWER** _____

**4** Evaluate $\Sigma_{j=1}^{4}\ j!/(j-1)!$.

**ANSWER** _____

**5** Evaluate 6!/(2!4!).

**ANSWER** _____

**6** Which is larger, $\Sigma_{i=1}^{3}\ i!$ or 4!/(2!2!)? Give both values.

**ANSWER** _____

_____

## APPLY IT Show your work

**1** An esports club gives three different game codes to three of its nine members. No member gets more than one code. Use factorials to find how many assignments are possible.

WORK SPACE

**ANSWER** _____

**2** A dance committee has 4 seniors and 5 juniors. It randomly chooses 2 members, with every pair equally likely. What is the probability that both chosen members are seniors? Use factorials to count the pairs.

WORK SPACE

**ANSWER** _____

## CHALLENGE One step up

An esports squad has 3 players from Team A and 4 from Team B. A committee of 3 is chosen at random, with every group equally likely. Write a sum using factorials to count committees with at least one player from each team. Then find the probability of choosing such a committee.

WORK SPACE AND ANSWER

# Count the Possibilities



## PART 1 · PRACTICE

| # | PROBLEM | ANSWER |
|---|---|---|
| 1 | Evaluate $\Sigma_{i=1}^{4}$ (2i-1). | **16** |
| 2 | Evaluate 6!/4!. | **30** |
| 3 | Evaluate $\Sigma_{k=1}^{3}$ k!. | **9** |
| 4 | Evaluate $\Sigma_{j=1}^{4}$ j!/(j-1)!. | **10** |
| 5 | Evaluate 6!/(2!4!). | **15** |
| 6 | Which is larger, $\Sigma_{i=1}^{3}$ i! or 4!/(2!2!)? Give both values. | **$\Sigma_{i=1}^{3}$ i! is larger: 9 > 6.** |

## PART 2 · APPLY IT

1. **9!/6! = 504 assignments** There are 9 choices for the first code, 8 for the second, and 7 for the third. Multiply to get 9!/6! = 504.
2. **(4!/(2!2!))/(9!/(2!7!)) = 6/36 = 1/6** Count 4!/(2!2!) = 6 pairs of seniors and 9!/(2!7!) = 36 possible pairs. The probability is 6/36 = 1/6.

## CHALLENGE

**$\Sigma_{k=1}^{2}$ [3!/(k!(3-k)!)] [4!/((3-k)!(1+k)!)] = 18 + 12 = 30 committees. There are 7!/(3!4!) = 35 committees in all, so the probability is 30/35 = 6/7.**

Accept equivalent expanded sums or products in place of the factorial expressions. For the challenge, students may add the two cases in either order.