LESSON MAKER PRO

Slope at an Instant

Use a limit to find a derivative at one input.

HIGH SCHOOL · C.A.10

C.A.10 "State and apply the formal definition of a derivative."

NAME _____ DATE _____

THE DERIVATIVE DEFINITION

The derivative f'(a) is the slope of a function at x=a. Its formal definition is f'(a) = lim as h approaches 0 of (f(a+h) - f(a))/h, if the limit exists. Substitute first, simplify the quotient when possible, and then take the limit.

WORKED EXAMPLE

Use the formal definition to find f'(5) for $f(x)=4x^2$.

1. f(5)=100, and $f(5+h)=4(5+h)^2=100+40h+4h^2$.
2. $(f(5+h)-f(5))/h=(40h+4h^2)/h=40+4h$ for $h\neq0$.
3. As h approaches 0, 40+4h approaches 40.

Answer: f'(5)=40

PRACTICE 6 problems

1 For $f(x)=x^2$, write the simplified difference quotient at x=1. Then use the formal definition to find f'(1).

ANSWER _____

2 For $f(x)=3x^2$, write the simplified difference quotient at x=-1. Then use the formal definition to find f'(-1).

ANSWER _____

3 For $f(x)=x^3$, write the simplified difference quotient at x=2. Then use the formal definition to find f'(2).

ANSWER _____

4 For f(x)=1/x, write the simplified difference quotient at x=-2. Then use the formal definition to find f'(-2).

ANSWER _____

5 For f(x)=6x+1, write the simplified difference quotient at x=0. Then use the formal definition to find f'(0).

ANSWER _____

6 For $f(x)=x^2-3x$, write the simplified difference quotient at x=-3. Then use the formal definition to find f'(-3).

ANSWER _____

APPLY IT Show your work

1 A game download has used $D(t)=t^2+6t$ megabytes after t seconds. Use the formal definition to find D'(1). What does your answer mean?

WORK SPACE

ANSWER _____

2 During the first 3 hours of use, a phone's battery level is $C(t)=90-3t^2$ percent, where t is hours since it was unplugged. Use the formal definition to find C'(1). What does the sign tell you?

WORK SPACE

ANSWER _____

CHALLENGE One step up

Let $f(x)=|x|$. Use the formal definition to decide whether f'(0) exists. Show what the difference quotient equals when $h>0$ and when $h<0$, then explain your decision.

WORK SPACE AND ANSWER

Slope at an Instant

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For $f(x)=x^2$, write the simplified difference quotient at x=1. Then use the formal definition to find f'(1).	**2+h; f'(1)=2**
2	For $f(x)=3x^2$, write the simplified difference quotient at x=-1. Then use the formal definition to find f'(-1).	**-6+3h; f'(-1)=-6**
3	For $f(x)=x^3$, write the simplified difference quotient at x=2. Then use the formal definition to find f'(2).	**$12+6h+h^2$; f'(2)=12**
4	For f(x)=1/x, write the simplified difference quotient at x=-2. Then use the formal definition to find f'(-2).	**1/[2(h-2)]; f'(-2)=-1/4**
5	For f(x)=6x+1, write the simplified difference quotient at x=0. Then use the formal definition to find f'(0).	**6; f'(0)=6**
6	For $f(x)=x^2-3x$, write the simplified difference quotient at x=-3. Then use the formal definition to find f'(-3).	**-9+h; f'(-3)=-9**

PART 2 · APPLY IT

1. **D'(1)=8 megabytes per second. At 1 second, the amount downloaded is increasing at a rate of 8 megabytes per second.** The difference quotient is (D(1+h)-D(1))/h=8+h. Its limit as h approaches 0 is 8.
2. **C'(1)=-6 percentage points per hour. The negative sign means the battery level is decreasing at 1 hour.** The difference quotient is (C(1+h)-C(1))/h=-6-3h. Its limit as h approaches 0 is -6.

CHALLENGE

The difference quotient is |h|/h. It equals 1 when h>0 and -1 when h<0. Because these approach different values, f'(0) does not exist.

Accept equivalent forms of the simplified difference quotients. Students may cancel h only when h≠0, which is valid while taking the limit as h approaches 0.