LESSON MAKER PRO

Derivative Detective

Differentiate functions inside functions and equations with y on both sides.

HIGH SCHOOL · C.A.12

C.A.12 "Use the chain rule and implicit differentiation."

NAME _____ DATE _____

TWO RULES TO USE

When one function is inside another, differentiate the outside function, then multiply by the derivative of the inside function. When x and y appear in the same equation, treat y as a function of x. Each time you differentiate a term involving y, include dy/dx, then solve for it.

WORKED EXAMPLE

Find dy/dx if $y = (13x - 17)^{11}$.

1. Differentiate the outside: $11(13x - 17)^{10}$.
2. Differentiate the inside: $d(13x - 17)/dx = 13$.
3. Multiply: $dy/dx = 11(13x - 17)^{10}(13)$.

Answer: $dy/dx = 143(13x - 17)^{10}$

PRACTICE 6 problems

1 Find dy/dx if $y = (2x + 5)^4$.

ANSWER _____

2 Find dy/dx if $y = (x^2 - 3x)^5$.

ANSWER _____

3 Find dy/dx if $y = 1/(4x - 7)^2$.

ANSWER _____

4 Find dy/dx if $x^2 + y^2 = 9$.

ANSWER _____

5 Find dy/dx if $x^2y + y^3 = 6$.

ANSWER _____

6 Find dy/dx if $xy^2 + 4x^2 = 8$.

ANSWER _____

APPLY IT Show your work

1 A video game's score is $S(t) = (t^2 + 2)^3$ points, where t is the number of minutes since a round began. How fast is the score changing at t = 2?

WORK SPACE

ANSWER _____

2 A drone's position on a level field follows $x^2 + y^2 = 25$, where x is meters east and y is meters north of a charging station. At the position (3, 4), what is dy/dx, the change in its north position per meter of change in its east position?

WORK SPACE

ANSWER _____

CHALLENGE One step up

The curve $(x^2 + y^2)^2 = 4xy$ passes through (1, 1). Find dy/dx at that point. Show how you use the chain rule on the left side.

WORK SPACE AND ANSWER

Derivative Detective

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find dy/dx if $y = (2x + 5)^4$.	**$8(2x + 5)^3$**
2	Find dy/dx if $y = (x^2 - 3x)^5$.	**$5(2x - 3)(x^2 - 3x)^4$**
3	Find dy/dx if $y = 1/(4x - 7)^2$.	**$-8/(4x - 7)^3$**
4	Find dy/dx if $x^2 + y^2 = 9$.	**$-x/y$**
5	Find dy/dx if $x^2y + y^3 = 6$.	**$-2xy/(x^2 + 3y^2)$**
6	Find dy/dx if $xy^2 + 4x^2 = 8$.	**$-(y^2 + 8x)/(2xy)$**

PART 2 · APPLY IT

1. **432 points per minute** $S'(t) = 3(t^2 + 2)^2(2t)$. At $t = 2$, $S'(2) = 3(6)^2(4) = 432$.
2. **-3/4 meter north per meter east** Implicit differentiation gives $2x + 2y(dy/dx) = 0$, so $dy/dx = -x/y$. At (3, 4), $dy/dx = -3/4$.

CHALLENGE

$2(x^2 + y^2)(2x + 2y(dy/dx)) = 4(y + x(dy/dx))$. At (1, 1), this becomes $8(1 + dy/dx) = 4(1 + dy/dx)$, so $dy/dx = -1$.

Accept algebraically equivalent derivatives. In the drone problem, -3/4 describes the change in north position for each meter of change in east position.