LESSON MAKER PRO

# Smooth or Sudden?

*Check what happens where two rules meet.*

**HIGH SCHOOL · C.A.13**

**C.A.13** "Describe the relationship between differentiability and continuity."

NAME _____ DATE _____

## THE BIG IDEA

A function is continuous at a point if its value matches what the function approaches from both sides. To be differentiable there, it must also be continuous and have the same slope from both sides.

## WORKED EXAMPLE

For $f(x) = 8x + 9$ when $x < 7$ and $f(x) = 6x + 23$ when $x \geq 7$, is f continuous and differentiable at $x = 7$?

1. From the left, $f(x)$ approaches $8(7) + 9 = 65$. From the right, $f(7) = 6(7) + 23 = 65$.
2. The values match, so f is continuous at $x = 7$.
3. The left-hand slope is 8, but the right-hand slope is 6.

**Answer: Continuous at x = 7, but not differentiable there.**

## PRACTICE 6 problems

**1** Let $f(x) = x + 2$ when $x < 0$ and $f(x) = 2 - x$ when $x \geq 0$. Is f continuous at $x = 0$? Is it differentiable there? Give the two slopes.

**ANSWER** _____

**2** Let $f(x) = x + 2$ when $x < 0$ and $f(x) = x + 4$ when $x \geq 0$. Is f continuous at $x = 0$? Is it differentiable there?

**ANSWER** _____

**3** Let $f(x) = x^2$ when $x < 2$ and $f(x) = 4x - 4$ when $x \geq 2$. Is f continuous and differentiable at $x = 2$? Give the two slopes.

**ANSWER** _____

**4** Let $f(x) = (x^2 - 4)/(x - 2)$ when $x \neq 2$ and $f(2) = 0$. Find the limit as x approaches 2. Is f differentiable at $x = 2$?

**ANSWER** _____

**5** Let $f(x) = x^2 + 2x + 3$ when $x < 0$ and $f(x) = kx + 3$ when $x \geq 0$. Find k so f is differentiable at $x = 0$.

**ANSWER** _____

**6** Let $f(x) = x^2 + 2x + 3$ when $x \neq 0$ and $f(0) = a$. Find a so f is differentiable at $x = 0$. Then find $f'(0)$.

**ANSWER** _____

## APPLY IT Show your work

**1** Your headphones charge according to $B(x) = 5x + 10$ for $0 \le x < 2$ and $B(x) = 20 + 3(x - 2)$ for $x \ge 2$. Here, x is minutes and B(x) is battery percent. At $x = 2$, is the battery level continuous? Does the charging rate change suddenly?

WORK SPACE

**ANSWER** _____

**2** In a racing game, a kart's distance is $d(x) = x^2 + 10x$ for $0 \le x < 2$ and $d(x) = 14x - 4$ for $x \ge 2$. Distance is in meters, and x is in seconds. At $x = 2$, is the distance continuous? Does the kart's speed change suddenly? Give the speed on each side.

WORK SPACE

**ANSWER** _____

## CHALLENGE One step up

Let $f(x) = ax + b$ when $x < 2$ and $f(x) = 3x^2 + 2x + 1$ when $x \ge 2$. Find a and b so f is both continuous and differentiable at $x = 2$. Show how each condition helps.

WORK SPACE AND ANSWER

# Smooth or Sudden?



## PART 1 · PRACTICE

| # | PROBLEM | ANSWER |
|---|---|---|
| 1 | Let f(x) = x + 2 when x < 0 and f(x) = 2 - x when x ≥ 0. Is f continuous at x = 0? Is it differentiable there? Give the two slopes. | **Continuous, but not differentiable. The left-hand slope is 1, and the right-hand slope is -1.** |
| 2 | Let f(x) = x + 2 when x < 0 and f(x) = x + 4 when x ≥ 0. Is f continuous at x = 0? Is it differentiable there? | **Neither. The left-hand value approaches 2, but f(0) = 4.** |
| 3 | Let f(x) = $x^2$ when x < 2 and f(x) = 4x - 4 when x ≥ 2. Is f continuous and differentiable at x = 2? Give the two slopes. | **Yes to both. Both sides meet at 4, and both one-sided slopes are 4.** |
| 4 | Let f(x) = ($x^2$ - 4)/(x - 2) when x ≠ 2 and f(2) = 0. Find the limit as x approaches 2. Is f differentiable at x = 2? | **The limit is 4. No, f is not differentiable at x = 2 because the limit does not equal f(2).** |
| 5 | Let f(x) = $x^2$ + 2x + 3 when x < 0 and f(x) = kx + 3 when x ≥ 0. Find k so f is differentiable at x = 0. | **k = 2** |
| 6 | Let f(x) = $x^2$ + 2x + 3 when x ≠ 0 and f(0) = a. Find a so f is differentiable at x = 0. Then find f'(0). | **a = 3 and f'(0) = 2** |

## PART 2 · APPLY IT

1. **The battery level is continuous at x = 2. The charging rate changes suddenly from 5 to 3 percentage points per minute, so B is not differentiable there.** Both rules give 20 percent at x = 2. Their slopes are 5 and 3, so the one-sided rates do not match.
2. **The distance is continuous at x = 2, and the speed does not change suddenly. The speed is 14 meters per second on each side, so d is differentiable there.** Both distance rules give 24 meters at x = 2. The slope of the first rule at x = 2 is 2(2) + 10 = 14, which matches the slope of the second rule.

## CHALLENGE

**The right-hand value at x = 2 is 17, so continuity requires 2a + b = 17. The right-hand slope there is 6(2) + 2 = 14, so differentiability requires a = 14. Therefore b = -11.**

Accept matching one-sided values as evidence of continuity and matching one-sided slopes as evidence of differentiability. A mismatch in the values rules out differentiability, even if the slopes on either side match.