LESSON MAKER PRO

Rates Right Now

Use derivatives to find and explain instantaneous change.

HIGH SCHOOL · C.A.14

C.A.14 "Define a derivative and explain the purpose/utility of the derivative."

NAME _____ DATE _____

THE BIG IDEA

The derivative f'(a) is the instantaneous rate of change of f at x=a, which is also the slope of the tangent line there. Find it by taking the limit of [f(a+h)-f(a)]/h as h approaches 0. For $f(x)=mx^2+nx+k$, the shortcut is f'(x)=2mx+n.

WORKED EXAMPLE

Use the limit definition to find f'(4) if $f(x)=x^2+11x$.

1. $f(4)=4^2+11(4)=60$.
2. $f(4+h)=(4+h)^2+11(4+h)=60+19h+h^2$.
3. $[f(4+h)-f(4)]/h=(19h+h^2)/h=19+h$.
4. As h approaches 0, 19+h approaches 19.

Answer: f'(4)=19.

PRACTICE 6 problems

1 Use the limit definition to find f'(2) if $f(x)=x^2$.

ANSWER _____

2 Find f'(0) if $f(x)=x^2+5x$.

ANSWER _____

3 If f(x)=9-2x, find f'(6). Is the function increasing or decreasing there?

ANSWER _____

4 Find f'(-1) if $f(x)=3x^2$.

ANSWER _____

5 For $f(x)=x^2-6x$, find the x-value where the instantaneous rate of change is 0.

ANSWER _____

6 For $f(x)=x^2-x$, find the instantaneous rate of change at x=2 and the average rate of change from x=0 to x=2.

ANSWER _____

APPLY IT Show your work

1 The number of views on a new video is $V(t)=t^2+8t$, measured in hundreds of views, where t is hours since posting. Find $V'(2)$ and explain what it means in views per hour.

WORK SPACE

ANSWER _____

2 A scooter's battery level is $B(t)=100-2t^2$ percent, where t is hours into a ride. Find $B'(3)$ and explain what its sign means.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Two accounts have follower counts $A(t)=t^2+2t$ and $B(t)=5t$, where t is weeks after they began posting. At $t=3$, which account is gaining followers faster? Explain why equal follower counts at that time do not answer the question.

WORK SPACE AND ANSWER

Rates Right Now

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Use the limit definition to find f'(2) if $f(x)=x^2$.	**f'(2)=4.**
2	Find f'(0) if $f(x)=x^2+5x$.	**f'(0)=5.**
3	If f(x)=9-2x, find f'(6). Is the function increasing or decreasing there?	**f'(6)=-2; the function is decreasing.**
4	Find f'(-1) if $f(x)=3x^2$.	**f'(-1)=-6.**
5	For $f(x)=x^2-6x$, find the x-value where the instantaneous rate of change is 0.	**x=3.**
6	For $f(x)=x^2-x$, find the instantaneous rate of change at x=2 and the average rate of change from x=0 to x=2.	**Instantaneous rate: 3. Average rate: 1.**

PART 2 · APPLY IT

1. **V'(2)=12 hundreds of views per hour. At 2 hours, views are increasing at an instantaneous rate of 1,200 views per hour.** V'(t)=2t+8, so V'(2)=12. Each unit of V represents 100 views.
2. **B'(3)=-12 percentage points per hour. At 3 hours, the battery level is falling.** B'(t)=-4t, so B'(3)=-12. The negative sign shows that battery level is decreasing at that instant.

CHALLENGE

Account A is gaining followers faster: A'(3)=8 followers per week, while B'(3)=5 followers per week. Both have 15 followers at t=3, but equal counts do not imply equal instantaneous rates of change.

Accept equivalent derivative calculations using either the limit definition or the quadratic shortcut. In context, require units and an interpretation of the derivative's sign.