LESSON MAKER PRO

Motion in Moments

Use derivatives to find rates at a specific time.

HIGH SCHOOL · C.A.15

C.A.15 "Apply the derivative as a rate of change in varied contexts, including velocity, speed, and acceleration."

NAME _____ DATE _____

RATES RIGHT NOW

The derivative gives a rate of change at a specific moment. For position s(t), velocity is s'(t), speed is |s'(t)|, and acceleration is the derivative of velocity. Include units with each rate.

WORKED EXAMPLE

A cart's position is $s(t) = 7t^2 + 11t$ meters, where t is in seconds. Find its velocity at $t = 5$.

1. Differentiate position: $v(t) = s'(t) = 14t + 11$.
2. Substitute $t = 5$: $v(5) = 14(5) + 11$.
3. Calculate: $v(5) = 81$.

Answer: 81 m/s

PRACTICE 6 problems

1 A robot's position is $s(t) = t^2 + 4t$ meters, where t is in seconds. Find its velocity at $t = 2$.

ANSWER _____

2 A ball's position is $s(t) = 20t - 2t^2$ meters, where t is in seconds. Find its velocity at $t = 4$.

ANSWER _____

3 A toy car's position is $s(t) = 16t - t^2$ meters, where t is in seconds. Find its velocity and speed at $t = 10$.

ANSWER _____

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4 A cyclist's velocity is $v(t) = 4t^2 - 6t$ meters per second, where t is in seconds. Find the acceleration at $t = 2$.

ANSWER _____

5 The amount of water in a tank is $Q(t) = 50 + 8t - t^2$ liters, where t is in minutes. How fast is the amount changing at $t = 2$?

ANSWER _____

6 A cart's position is $s(t) = t^2 - 8t$ meters, where t is in seconds. At what time is the cart momentarily still?

ANSWER _____

APPLY IT Show your work

1 At a skate park, a skater's position along a straight path is $s(t) = 24t - 2t^2$ meters, where t is in seconds. Find the skater's velocity and acceleration at $t = 4$.

WORK SPACE

ANSWER _____

2 During a gaming session, a phone's battery level is $B(t) = 100 - 6t - t^2$ percent, where t is in hours. At $t = 2$, how fast is the battery level changing? State whether it is rising or falling.

WORK SPACE

ANSWER _____

CHALLENGE One step up

A small robot moves along a straight track. Its position is $s(t) = t^3 - 6t^2 + 9t$ meters for $0 \le t \le 4$ seconds. At what times does it change direction, and what total distance does it travel from $t = 0$ to $t = 4$?

WORK SPACE AND ANSWER

Motion in Moments

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	A robot's position is $s(t) = t^2 + 4t$ meters, where t is in seconds. Find its velocity at t = 2.	**8 m/s**
2	A ball's position is $s(t) = 20t - 2t^2$ meters, where t is in seconds. Find its velocity at t = 4.	**4 m/s**
3	A toy car's position is $s(t) = 16t - t^2$ meters, where t is in seconds. Find its velocity and speed at t = 10.	**Velocity: -4 m/s; speed: 4 m/s**
4	A cyclist's velocity is $v(t) = 4t^2 - 6t$ meters per second, where t is in seconds. Find the acceleration at t = 2.	**10 m/s²**
5	The amount of water in a tank is $Q(t) = 50 + 8t - t^2$ liters, where t is in minutes. How fast is the amount changing at t = 2?	**4 L/min**
6	A cart's position is $s(t) = t^2 - 8t$ meters, where t is in seconds. At what time is the cart momentarily still?	**t = 4 seconds**

PART 2 · APPLY IT

1. **Velocity: 8 m/s; acceleration: -4 m/s²** Differentiate to get $v(t) = 24 - 4t$ and $a(t) = -4$. At t = 4, velocity is 8 m/s.
2. **-10 percentage points per hour; the battery level is falling.** $B'(t) = -6 - 2t$, so $B'(2) = -10$ percentage points per hour. The negative rate means the battery level is falling.

CHALLENGE

It changes direction at t = 1 second and t = 3 seconds. Total distance: 12 meters.

Accept equivalent unit notation. For the challenge, $v(t) = 3(t - 1)(t - 3)$ changes sign at both times, and the positions at t = 0, 1, 3, and 4 are 0, 4, 0, and 4 meters.