LESSON MAKER PRO

Lines That Touch

Use derivatives to find tangent and normal lines.

HIGH SCHOOL · C.A.16

C.A.16 "Apply the derivative to find tangent lines and normal lines to given curves at given points."

NAME _____ DATE _____

FROM SLOPE TO LINE

At x=a, the derivative f'(a) gives the tangent line's slope, and the point on the curve is (a, f(a)). If f'(a) is not zero, the normal line's slope is -1/f'(a). Use y-f(a)=m(x-a) to write either line. If the tangent is horizontal, its normal is the vertical line x=a.

WORKED EXAMPLE

Find the tangent and normal lines to $y=x^2+4x+6$ at x=-1.

1. Differentiate: f'(x)=2x+4.
2. Find the point and slope: f(-1)=3 and f'(-1)=2, so the point is (-1, 3).
3. Tangent slope: 2. Write y-3=2(x+1).
4. Normal slope: -1/2. Write y-3=-1/2(x+1).

Answer: Tangent: y-3=2(x+1). Normal: y-3=-1/2(x+1).

PRACTICE 6 problems

1 Find the tangent line to $y=x^2+2x+3$ at x=0.

ANSWER _____

2 Find the normal line to $y=x^2-4x+1$ at x=3.

ANSWER _____

3 Find the tangent and normal lines to $y=2x^2-3x+1$ at x=2.

ANSWER _____

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4 Find the normal line to $y=x^3-2x$ at x=1.

ANSWER _____

5 Find the tangent and normal lines to $y=x^2-6x+4$ at x=3.

ANSWER _____

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6 Find the tangent line to $y=3x^2-x+2$ at x=1.

ANSWER _____

APPLY IT Show your work

1 A creator models a video's viewers, in hundreds, x hours after posting with $V(x)=-x^2+6x+10$ for the first few hours. Find the tangent line at x=2, using y for viewers in hundreds. What does its slope mean?

WORK SPACE

ANSWER _____

2 A skate park designer models a ramp's side profile with $y=1/2x^2+x$, where x and y are in meters. At x=2, a support meets the ramp at a right angle. Find the equation of the line containing the support at that point.

WORK SPACE

ANSWER _____

CHALLENGE One step up

For $y=x^3-3x$, find every point where the tangent is parallel to y=9x-4. Find the normal line at each point.

WORK SPACE AND ANSWER

Lines That Touch

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the tangent line to $y=x^2+2x+3$ at $x=0$.	**$y=2x+3$**
2	Find the normal line to $y=x^2-4x+1$ at $x=3$.	**$y+2=-1/2(x-3)$**
3	Find the tangent and normal lines to $y=2x^2-3x+1$ at $x=2$.	**Tangent: $y-3=5(x-2)$. Normal: $y-3=-1/5(x-2)$.**
4	Find the normal line to $y=x^3-2x$ at $x=1$.	**$y=-x$**
5	Find the tangent and normal lines to $y=x^2-6x+4$ at $x=3$.	**Tangent: $y=-5$. Normal: $x=3$.**
6	Find the tangent line to $y=3x^2-x+2$ at $x=1$.	**$y-4=5(x-1)$**

PART 2 · APPLY IT

1. **$y=2x+14$. At 2 hours, the model's viewer count is increasing at 200 viewers per hour.** $V(2)=18$ and $V'(x)=-2x+6$, so $V'(2)=2$ hundreds of viewers per hour. The line through (2, 18) with slope 2 is $y-18=2(x-2)$.
2. **$y-4=-1/3(x-2)$** The support meets the ramp at (2, 4). The ramp's slope there is $y'=x+1=3$, so the perpendicular support has slope -1/3.

CHALLENGE

The points are (-2, -2) and (2, 2). Their normal lines are $y+2=-1/9(x+2)$ and $y-2=-1/9(x-2)$, respectively.

Accept equivalent line equations. For the viewer model, accept an interpretation that specifies 200 viewers per hour at $x=2$, rather than a constant rate for all hours.