LESSON MAKER PRO

Slope Signals

Use derivatives to predict how functions change.

HIGH SCHOOL · C.A.17

C.A.17 "Predict and explain the relationships between functions and their derivatives."

NAME _____ DATE _____

WHAT THE DERIVATIVE TELLS YOU

To differentiate a term such as ax^n, multiply its coefficient by n and reduce the exponent by 1. A positive derivative means the function is increasing at that input, while a negative derivative means it is decreasing. A derivative of zero gives a horizontal tangent, but you must check the signs on both sides to decide whether there is a maximum or minimum.

WORKED EXAMPLE

For $f(x) = 7x^2 - 11x + 13$, find $f'(5)$ and predict whether f is increasing or decreasing at $x = 5$.

1. Differentiate each term: $f'(x) = 14x - 11$.
2. Substitute $x = 5$: $f'(5) = 14(5) - 11 = 59$.
3. Since 59 is positive, f is increasing at $x = 5$.

Answer: f'(5) = 59; increasing.

PRACTICE 6 problems

1 For $f(x) = x^2 + 2x$, find $f'(0)$.

ANSWER _____

2 For $f(x) = 3x^2 - 6x$, find $f'(2)$. Is f increasing or decreasing there?

ANSWER _____

3 For $f(x) = -x^2 + 8x$, find $f'(6)$. Is f increasing or decreasing there?

ANSWER _____

4 For $f(x) = x^3 - 3x$, find the slope of the tangent line at $x = 0$.

ANSWER _____

5 For $f(x) = -2x^2 + 8x$, find $f'(2)$. Does f have a local maximum or minimum at $x = 2$?

ANSWER _____

6 For $f(x) = x^2 + 6x + 8$, find $f'(-3)$. Does f have a local maximum or minimum at $x = -3$?

ANSWER _____

APPLY IT Show your work

1 The number of people watching a live stream is $F(x) = -x^2 + 8x + 12$, where x is hours after the stream starts and $0 \le x \le 6$. Find $F'(2)$. What does its sign tell you about the audience then?

WORK SPACE

ANSWER _____

2 A scooter's battery charge is $B(x) = 80 - 3x^2$ percent, where x is hours into a ride and $0 \le x \le 4$. Find $B'(1)$. Explain what it means for the battery at that time.

WORK SPACE

ANSWER _____

CHALLENGE One step up

For $f(x) = 3x^2 - 12x + 8$, find where f has a local minimum. Explain how the sign of $f'(x)$ changes on either side of that input.

WORK SPACE AND ANSWER

Slope Signals

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For $f(x) = x^2 + 2x$, find $f'(0)$.	**2**
2	For $f(x) = 3x^2 - 6x$, find $f'(2)$. Is f increasing or decreasing there?	**f'(2) = 6; increasing.**
3	For $f(x) = -x^2 + 8x$, find $f'(6)$. Is f increasing or decreasing there?	**f'(6) = -4; decreasing.**
4	For $f(x) = x^3 - 3x$, find the slope of the tangent line at $x = 0$.	**-3**
5	For $f(x) = -2x^2 + 8x$, find $f'(2)$. Does f have a local maximum or minimum at $x = 2$?	**f'(2) = 0; local maximum.**
6	For $f(x) = x^2 + 6x + 8$, find $f'(-3)$. Does f have a local maximum or minimum at $x = -3$?	**f'(-3) = 0; local minimum.**

PART 2 · APPLY IT

1. **F'(2) = 4 viewers per hour; the audience is growing.** F'(x) = -2x + 8, so F'(2) = 4. The positive rate means the number of viewers is increasing.
2. **B'(1) = -6 percentage points per hour; the battery charge is falling.** B'(x) = -6x, so B'(1) = -6. At that moment, the battery percentage is decreasing at a rate of 6 percentage points per hour.

CHALLENGE

f'(x) = 6x - 12, which is zero at x = 2. It is negative for x < 2 and positive for x > 2, so f changes from decreasing to increasing and has a local minimum at x = 2.

Accept equivalent wording for increasing and decreasing. For the battery problem, the rate is measured in percentage points per hour, not percent of the current charge per hour.