LESSON MAKER PRO

Peak Value Quest

Find the highest or lowest value a quadratic function can reach.

HIGH SCHOOL · C.A.19

C.A.19 "Solve optimization problems."

NAME _____ DATE _____

FIND THE VERTEX

For $f(x) = ax^2 + bx + c$, the vertex occurs at $x = -b/(2a)$. Substitute that x-value into the function to find the maximum when a is negative or the minimum when a is positive. If x has a limited range, check whether the vertex is allowed and compare it with the endpoints.

WORKED EXAMPLE

Find the maximum value of $f(x) = -7x^2 + 42x + 13$.

1. Use $a = -7$ and $b = 42$: $x = -42/(2(-7)) = 3$.
2. Substitute: $f(3) = -7(3^2) + 42(3) + 13 = 76$.
3. Because a is negative, the vertex gives the maximum.

Answer: The maximum value is 76.

PRACTICE 6 problems

1 Find the maximum value of $f(x) = -x^2 + 8x + 2$.

ANSWER _____

2 Find the minimum value of $g(x) = 2x^2 - 20x + 5$.

ANSWER _____

3 At what x-value does $h(x) = -2x^2 + 12x - 1$ reach its maximum?

ANSWER _____

4 Find the minimum value of $k(x) = x^2 + 8x + 4$.

ANSWER _____

5 Find both the x-value and the maximum value of $p(x) = -4x^2 + 8x + 9$.

ANSWER _____

6 For $0 <= x <= 4$, find both the x-value and the maximum value of $q(x) = -x^2 + 10x + 1$.

ANSWER _____

APPLY IT Show your work

1 A school dance expects to sell 100 tickets at $6 each. For each $1 increase in ticket price, it expects to sell 5 fewer tickets. Let x be the number of $1 increases. What ticket price gives the greatest revenue, and what is that revenue?

WORK SPACE

ANSWER _____

2 A gaming club sells 20 sticker packs at $12 per pack. It expects to sell 5 more packs for each $1 discount. Let x be the number of $1 discounts. What price per pack gives the greatest revenue, and what is that revenue?

WORK SPACE

ANSWER _____

CHALLENGE One step up

A gaming club models its daily net earnings, in dollars, by $E(x) = -2x^2 + 22x - 15$, where x is the number of snack bundles sold. It can sell only whole bundles, from 0 to 10. What are the greatest possible net earnings, and how many bundles give that amount? Explain your reasoning.

WORK SPACE AND ANSWER

Peak Value Quest

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the maximum value of $f(x) = -x^2 + 8x + 2$.	**18**
2	Find the minimum value of $g(x) = 2x^2 - 20x + 5$.	**-45**
3	At what x-value does $h(x) = -2x^2 + 12x - 1$ reach its maximum?	**x = 3**
4	Find the minimum value of $k(x) = x^2 + 8x + 4$.	**-12**
5	Find both the x-value and the maximum value of $p(x) = -4x^2 + 8x + 9$.	**The maximum value is 13 at x = 1.**
6	For $0 <= x <= 4$, find both the x-value and the maximum value of $q(x) = -x^2 + 10x + 1$.	**The maximum value is 25 at x = 4.**

PART 2 · APPLY IT

1. **A $13 ticket price gives the greatest revenue, $845.** Revenue is $R(x) = (6 + x)(100 - 5x) = -5x^2 + 70x + 600$. Its vertex is at x = 7, so the price is $13 and the revenue is 13(65) = $845.
2. **A price of $8 per pack gives the greatest revenue, $320.** Revenue is $R(x) = (12 - x)(20 + 5x) = -5x^2 + 40x + 240$. Its vertex is at x = 4, giving a price of $8 and revenue of 8(40) = $320.

CHALLENGE

The greatest possible net earnings are $45, reached by selling either 5 or 6 bundles. The vertex is at x = 5.5, which is not a whole number. The nearest allowed whole numbers, 5 and 6, both give E(x) = 45.

Accept equivalent vertex calculations. For the challenge, the explanation should check both whole numbers nearest the vertex.