LESSON MAKER PRO

Rates and Totals

Use integrals to connect changing rates with accumulated amounts.

HIGH SCHOOL · C.A.20

C.A.20 "State and apply the First and Second Fundamental Theorem of Calculus."

NAME _____ DATE _____

TWO WAYS TO USE AN INTEGRAL

If f is continuous and $A(x) = \int_a^x f(t)\,dt$, the First Fundamental Theorem of Calculus says $A'(x) = f(x)$. If $F'(x) = f(x)$, the Second Fundamental Theorem of Calculus says $\int_a^b f(x)\,dx = F(b) - F(a)$.

WORKED EXAMPLE

Evaluate $\int_8^{10} 6t\,dt$.

1. Find an antiderivative: $F(t) = 3t^2$.
2. Evaluate at the bounds: $F(10) = 300$ and $F(8) = 192$.
3. Subtract: $F(10) - F(8) = 300 - 192 = 108$.

Answer: 108

PRACTICE 6 problems

1 Evaluate $\int_0^2 3x^2\,dx$.

ANSWER _____

2 Evaluate $\int_{-1}^2 (2x + 3)\,dx$.

ANSWER _____

3 Evaluate $\int_0^3 (x^2 - 2x)\,dx$.

ANSWER _____

4 Find $d/dx\left[\int_0^x (t^2 + 2t)\,dt\right]$.

ANSWER _____

5 Find $d/dx\left[\int_{-2}^x (3t^2 - t)\,dt\right]$.

ANSWER _____

6 Find $d/dx\left[\int_x^0 (t^2 + 3)\,dt\right]$.

ANSWER _____

APPLY IT Show your work

1 A phone charges at a rate of $3t + 2$ percentage points per minute, where t is the number of minutes since you plugged it in. How many percentage points does its charge gain during the first 2 minutes?

WORK SPACE

ANSWER _____

2 After x minutes, a video upload has used $D(x) = \int_0^x (t^2 + 2)\,dt$ megabytes of data. What is its data-use rate at $x = 3$ minutes?

WORK SPACE

ANSWER _____

CHALLENGE One step up

Let $A(u) = \int_0^u (2t^2 + 1)\,dt$. Find $d/dx\,[A(x^2)]$. Explain why your answer has an extra factor.

WORK SPACE AND ANSWER

Rates and Totals

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Evaluate $\int_0^2 3x^2\,dx$.	**8**
2	Evaluate $\int_{-1}^2 (2x + 3)\,dx$.	**12**
3	Evaluate $\int_0^3 (x^2 - 2x)\,dx$.	**0**
4	Find $d/dx\,[\int_0^x (t^2 + 2t)\,dt]$.	**$x^2 + 2x$**
5	Find $d/dx\,[\int_{-2}^x (3t^2 - t)\,dt]$.	**$3x^2 - x$**
6	Find $d/dx\,[\int_x^0 (t^2 + 3)\,dt]$.	**$-x^2 - 3$**

PART 2 · APPLY IT

1. **10 percentage points** Integrate the charging rate: $\int_0^2 (3t + 2)\,dt = [(3/2)t^2 + 2t]$ from 0 to 2 = 10.
2. **11 megabytes per minute** By the First Fundamental Theorem, $D'(x) = x^2 + 2$. Thus $D'(3) = 11$ megabytes per minute.

CHALLENGE

$d/dx\,[A(x^2)] = (2x^4 + 1)(2x) = 4x^5 + 2x$. The chain rule adds the factor 2x because the input to A is x^2.

Accept equivalent unsimplified expressions for derivatives. In problem 6, the negative sign is required because x is the lower bound.