LESSON MAKER PRO

Reverse the Rate

Use the power rule and substitution to find antiderivatives.

HIGH SCHOOL · C.A.21

C.A.21 "Apply the power rule and u-substitution to evaluate indefinite integrals."

NAME _____ DATE _____

TWO MOVES FOR INTEGRALS

For $n \neq -1$, $\int x^n \, dx = x^{n+1}/(n+1) + C$. When a power contains an expression inside it, let u equal that expression and account for its derivative before using the power rule.

WORKED EXAMPLE

Evaluate $\int 22(11x + 13)^8 \, dx$.

1. Let $u = 11x + 13$, so $du = 11 \, dx$.
2. Rewrite the integral as $2\int u^8 \, du$.
3. Apply the power rule: $2u^9/9 + C$.

Answer: $2(11x + 13)^9/9 + C$

PRACTICE 6 problems

1 Evaluate $\int x^4 \, dx$.

ANSWER _____

2 Evaluate $\int 5x^2 \, dx$.

ANSWER _____

3 Evaluate $\int x^{-3} \, dx$.

ANSWER _____

4 Evaluate $\int 4(2x + 1)^3 \, dx$.

ANSWER _____

5 Evaluate $\int 6x(x^2 + 3)^4 \, dx$.

ANSWER _____

6 Evaluate $\int 10x^4/(2x^5 + 1)^2 \, dx$.

ANSWER _____

APPLY IT Show your work

1 In a game, you earn coins at a rate of $12x(3x^2 + 2)^2$ coins per minute, where x is minutes since the round began. You start with 20 coins. Find your coin balance after x minutes.

WORK SPACE

ANSWER _____

2 A music track gains streams at a rate of $5(5x + 1)^3$ streams per day, where x is days after release. It has 0 streams at $x = 0$. Find a formula for its total streams after x days.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Evaluate $\int [6x^2 + 4x(2x^2 + 1)^3]\, dx$.

WORK SPACE AND ANSWER

Reverse the Rate

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Evaluate $\int x^4\,dx$.	$\mathbf{x^5/5 + C}$
2	Evaluate $\int 5x^2\,dx$.	$\mathbf{5x^3/3 + C}$
3	Evaluate $\int x^{-3}\,dx$.	$\mathbf{-1/(2x^2) + C}$
4	Evaluate $\int 4(2x + 1)^3\,dx$.	$\mathbf{(2x + 1)^4/2 + C}$
5	Evaluate $\int 6x(x^2 + 3)^4\,dx$.	$\mathbf{3(x^2 + 3)^5/5 + C}$
6	Evaluate $\int 10x^4/(2x^5 + 1)^2\,dx$.	$\mathbf{-1/(2x^5 + 1) + C}$

PART 2 · APPLY IT

1. **$20 + 2[(3x^2 + 2)^3 - 8]/3$ coins** Let $u = 3x^2 + 2$, so $du = 6x\,dx$. Integrate the rate to get $2u^3/3$, then subtract its value at $x = 0$ and add the starting 20 coins.
2. **$[(5x + 1)^4 - 1]/4$ streams** Let $u = 5x + 1$, so $du = 5\,dx$. The antiderivative is $u^4/4 + C$, and the initial total of 0 makes $C = -1/4$.

CHALLENGE

$2x^3 + (2x^2 + 1)^4/4 + C$

Accept algebraically equivalent antiderivatives that differ by a constant. For the word problems, accept equivalent formulas that satisfy the stated starting totals.