LESSON MAKER PRO

Approaching the Edge

Find what a function approaches from its graph, table, or rule.

HIGH SCHOOL · C.A.5

C.A.5 "Investigate, describe, and determine asymptotic behavior using tables, graphs, and analytical methods"

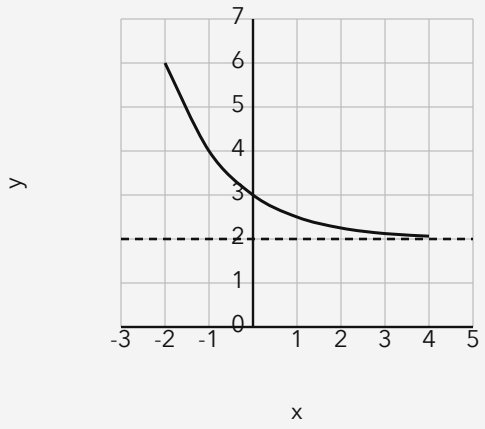
NAME _____ **DATE** _____

THE BIG IDEA

A horizontal asymptote is a line y = a that a graph gets closer to as x moves far left or far right. A vertical asymptote is a line x = a that a graph approaches while its y-values grow without bound. Use a graph or table to spot a pattern, then use the rule to check it.

WORKED EXAMPLE

The graph shows $y = 2 + (1/2)^x$. Find its horizontal asymptote as x increases.

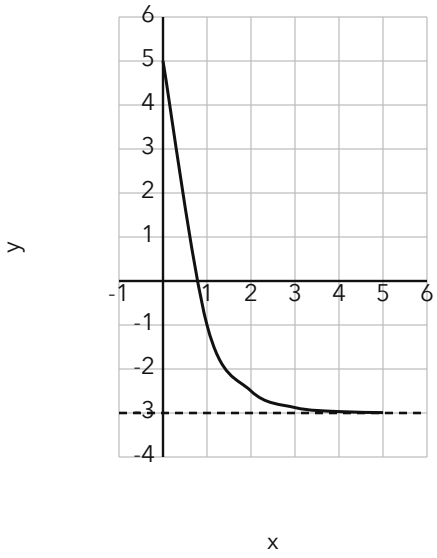


1. Read points moving right: the y-values go from 3 to 5/2 to 9/4.
2. These values get closer to 2.
3. Also, $(1/2)^x$ gets closer to 0 as x increases, so y gets closer to 2.

Answer: The horizontal asymptote is y = 2.

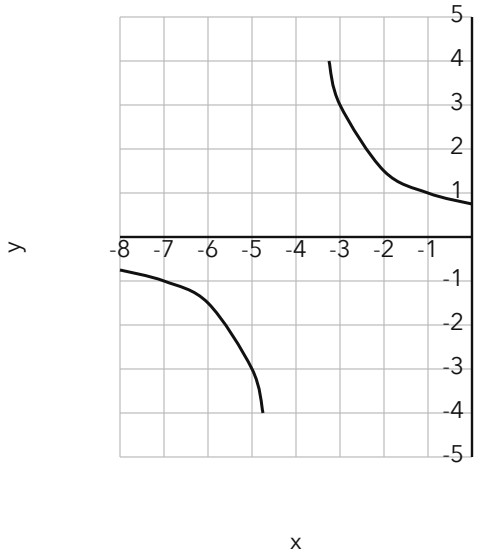
PRACTICE 6 problems

1 Look at the curve. What is its horizontal asymptote as x increases?



ANSWER _____

2 Look at both branches of the graph. Which vertical line do they approach?



ANSWER _____

3 Use the table and the rule f(x) = 6 - 8/(x + 1). Find the horizontal asymptote as x increases.

VALUES OF F(X)

x	f(x)
0	-2
1	2
3	4
7	5
15	11/2

ANSWER _____

4 Find the vertical and horizontal asymptotes of y = -5 + 7/(x - 3).

ANSWER _____

5 Find the vertical and horizontal asymptotes of y = (5x + 7)/(x - 6).

ANSWER _____

6 For $y = 9(2/3)^x - 4$, what happens to y as x increases without bound? What happens as x decreases without bound?

ANSWER _____

APPLY IT Show your work

1 A ticket app charges $15 per ticket plus a $9 fee per order. If you buy x tickets in one order, the average cost per ticket is C(x) = 15 + 9/x dollars. What does the average cost approach as x increases? Name the horizontal asymptote.

WORK SPACE

ANSWER _____

2 The time to edit a short video after x practice sessions is modeled by M(x) = 5 + 30/(x + 3) minutes, where x is at least 0. Find the horizontal asymptote as x increases, then find the predicted time after 7 sessions.

WORK SPACE

ANSWER _____

CHALLENGE One step up

A student says x = 3 is a vertical asymptote of $f(x) = (x^2 - 9)/(x - 3)$ because the denominator is 0 there. Is the student correct? Explain what happens to f(x) as x approaches 3.

WORK SPACE AND ANSWER

Approaching the Edge

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Look at the curve. What is its horizontal asymptote as x increases?	**y = -3**
2	Look at both branches of the graph. Which vertical line do they approach?	**x = -4**
3	Use the table and the rule f(x) = 6 - 8/(x + 1). Find the horizontal asymptote as x increases.	**y = 6**
4	Find the vertical and horizontal asymptotes of y = -5 + 7/(x - 3).	**Vertical: x = 3. Horizontal: y = -5.**
5	Find the vertical and horizontal asymptotes of y = (5x + 7)/(x - 6).	**Vertical: x = 6. Horizontal: y = 5.**
6	For $y = 9(2/3)^x - 4$, what happens to y as x increases without bound? What happens as x decreases without bound?	**As x increases, y approaches -4. As x decreases, y increases without bound.**

PART 2 · APPLY IT

1. **The average cost approaches $15 per ticket. The horizontal asymptote is y = 15.** As x increases, 9/x approaches 0, so C(x) approaches 15.
2. **The horizontal asymptote is y = 5. After 7 sessions, the predicted time is 8 minutes.** As x increases, 30/(x + 3) approaches 0. Substituting x = 7 gives M(7) = 5 + 30/10 = 8.

CHALLENGE

No. For x not equal to 3, f(x) = (x - 3)(x + 3)/(x - 3) = x + 3. As x approaches 3, f(x) approaches 6. The graph has a hole at (3, 6), not a vertical asymptote.

For the graph questions, accept a correctly written asymptote equation. In the challenge, the explanation must distinguish a hole from unbounded behavior.