LESSON MAKER PRO

Where Paths Meet

Check what happens as a function reaches a point.

HIGH SCHOOL · C.A.6

C.A.6 "Determine and justify the continuity and discontinuity of functions"

NAME _____ DATE _____

THE CONTINUITY CHECK

A function is continuous at a point when its left-hand limit, right-hand limit, and value at the point all exist and are equal. If any one is missing or different, the function is discontinuous there.

WORKED EXAMPLE

For x < 7, f(x) = x + 9. For x >= 7, f(x) = 3x - 5. Is f continuous at x = 7?

1. From the left, f(x) approaches 7 + 9 = 16.
2. From the right, f(x) approaches 3(7) - 5 = 16.
3. At the point, f(7) = 3(7) - 5 = 16.

Answer: Yes. Both one-sided limits and f(7) equal 16.

PRACTICE 6 problems

1 For x < 0, f(x) = 2x + 4. For x >= 0, f(x) = x + 4. Is f continuous at x = 0? Justify your answer.

ANSWER _____

2 For x < 2, f(x) = x + 6. For x >= 2, f(x) = 2x + 6. Is f continuous at x = 2? Justify your answer.

ANSWER _____

3 For x != 2, f(x) = $(x^2 - 4)/(x - 2)$. Let f(2) = 4. Is f continuous at x = 2? Justify your answer.

ANSWER _____

4 For x != 1, f(x) = $(x^2 - 1)/(x - 1)$. Let f(1) = 0. Is f continuous at x = 1? Justify your answer.

ANSWER _____

5 Let f(x) = 1/(x + 2). Is f continuous at x = -2? Justify your answer.

ANSWER _____

6 For x < 6, f(x) = x - 2. For x >= 6, f(x) = 2x - 8. Is f continuous at x = 6? Justify your answer.

ANSWER _____

APPLY IT Show your work

1 An online store charges $6 shipping when an order costs less than $40. Shipping is free for orders of $40 or more. Let T(x) be the total paid for an order costing x dollars before shipping. Is T continuous at x = 40? Justify your answer.

WORK SPACE

ANSWER _____

2 A bike rental costs 2x + 4 dollars for 0 <= x < 4 hours and x + 8 dollars for x >= 4 hours. Is the cost continuous at x = 4? Justify your answer.

WORK SPACE

ANSWER _____

CHALLENGE One step up

For $x < 0$, $f(x) = (x^2 + 2x)/x$. Let $f(0) = a$. For $x > 0$, $f(x) = bx + 2$. Find every value of a and b that makes f continuous at x = 0. Explain why.

WORK SPACE AND ANSWER

Where Paths Meet

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For x < 0, f(x) = 2x + 4. For x >= 0, f(x) = x + 4. Is f continuous at x = 0? Justify your answer.	**Yes. Both one-sided limits and f(0) equal 4.**
2	For x < 2, f(x) = x + 6. For x >= 2, f(x) = 2x + 6. Is f continuous at x = 2? Justify your answer.	**No. The left-hand limit is 8, but the right-hand limit and f(2) are 10.**
3	For x != 2, f(x) = $(x^2 - 4)/(x - 2)$. Let f(2) = 4. Is f continuous at x = 2? Justify your answer.	**Yes. For x != 2, f(x) = x + 2, so the limit at 2 is 4, which equals f(2).**
4	For x != 1, f(x) = $(x^2 - 1)/(x - 1)$. Let f(1) = 0. Is f continuous at x = 1? Justify your answer.	**No. For x != 1, f(x) = x + 1, so the limit at 1 is 2, but f(1) = 0.**
5	Let f(x) = 1/(x + 2). Is f continuous at x = -2? Justify your answer.	**No. f(-2) is undefined, and the function has no finite limit there.**
6	For x < 6, f(x) = x - 2. For x >= 6, f(x) = 2x - 8. Is f continuous at x = 6? Justify your answer.	**Yes. Both one-sided limits and f(6) equal 4.**

PART 2 · APPLY IT

1. **No. As x approaches 40 from below, T(x) approaches $46. At $40 and just above it, T(x) is $40 or approaches $40.** Use T(x) = x + 6 below $40 and T(x) = x at or above $40. The one-sided limits are $46 and $40, so they do not match.
2. **Yes. Both one-sided limits and the cost at 4 hours equal $12.** The first rule approaches 2(4) + 4 = 12, and the second rule approaches 4 + 8 = 12. The second rule also gives the cost at 4 hours.

CHALLENGE

a = 2, and b can be any real number. For x < 0, the expression simplifies to x + 2, so the left-hand limit is 2. For any real b, the right-hand limit of bx + 2 is also 2. Continuity requires f(0) = a = 2.

Accept equivalent justifications that correctly compare both one-sided limits with the function value. In the challenge, b is unrestricted because its term approaches 0 as x approaches 0.