LESSON MAKER PRO

Rates in Motion

Find instantaneous rates with exponential, logarithmic, and trigonometric functions.

HIGH SCHOOL · C.A.7

C.A.7 "Solve mathematical situations and application problems involving or using derivatives, including exponential, logarithmic, and trigonometric functions."

NAME _____ DATE _____

FIND THE RATE

A derivative gives the rate a function is changing at one input. Use $(e^x)' = e^x$, $(\ln x)' = 1/x$ for $x > 0$, and $(\sin x)' = \cos x$, with angles in radians. When x is replaced by an expression, multiply by the derivative of that expression.

WORKED EXAMPLE

Find $f'(x)$ if $f(x) = 7e^{11x^2 + 13}$.

1. The derivative of the exponent $11x^2 + 13$ is $22x$.
2. Multiply the original exponential expression by $22x$.
3. $f'(x) = 7e^{11x^2 + 13}(22x)$.

Answer: $f'(x) = 154xe^{11x^2 + 13}$

PRACTICE 6 problems

1 Find $f'(x)$ if $f(x) = 3e^{2x}$.

ANSWER _____

2 Find $f'(x)$ if $f(x) = 5 \ln x$, where $x > 0$.

ANSWER _____

3 Find $f'(x)$ if $f(x) = 4 \sin(3x)$.

ANSWER _____

4 Find the rate of change of $f(x) = e^{4x}$ at $x = 0$.

ANSWER _____

5 Find the slope of $f(x) = \ln x$ at $x = 2$.

ANSWER _____

6 Find the rate of change of $f(x) = 2 \sin x$ at $x = \pi/3$.

ANSWER _____

APPLY IT Show your work

1 A model for the number of shares of a new video is $S(t) = 40e^{t/2}$, where t is hours since posting. According to the model, how fast is the share count growing when the video is posted, at t = 0? Include units.

WORK SPACE

ANSWER _____

2 An app tracks how many minutes a student has spent using a study feature. Its model is $M(t) = 100 \ln(t + 1)$, where t is weeks since the feature launched. How fast is the modeled total changing at t = 3 weeks? Include units.

WORK SPACE

ANSWER _____

CHALLENGE One step up

For $f(x) = 2 \sin x + 3x$, find every x from 0 to 2pi, inclusive, where the slope is 4. Show how you know you found every value.

WORK SPACE AND ANSWER

Rates in Motion

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find f'(x) if f(x) = $3e^{2x}$.	**$6e^{2x}$**
2	Find f'(x) if f(x) = 5 ln x, where x > 0.	**5/x**
3	Find f'(x) if f(x) = 4 sin(3x).	**12 cos(3x)**
4	Find the rate of change of f(x) = e^{4x} at x = 0.	**4**
5	Find the slope of f(x) = ln x at x = 2.	**1/2**
6	Find the rate of change of f(x) = 2 sin x at x = pi/3.	**1**

PART 2 · APPLY IT

1. **20 shares per hour** S'(t) = $20e^{t/2}$. At t = 0, S'(0) = 20 shares per hour.
2. **25 minutes per week** M'(t) = 100/(t + 1). At t = 3, M'(3) = 100/4 = 25 minutes per week.

CHALLENGE

f'(x) = 2 cos x + 3. Setting the slope equal to 4 gives cos x = 1/2. On 0 ≤ x ≤ 2pi, this occurs only at x = pi/3 and x = 5pi/3.

Accept equivalent derivative expressions, including unsimplified products. The challenge requires both values in the stated interval.