LESSON MAKER PRO

Limits That Simplify

Find limits by rewriting expressions.

HIGH SCHOOL · C.A.8

C.A.8 "Calculate limits using algebraic methods."

NAME _____ DATE _____

SIMPLIFY, THEN SUBSTITUTE

If substitution gives 0/0, rewrite the expression before finding the limit. Factor and cancel a common factor, or multiply by a conjugate when a square root is involved. Then substitute the value x approaches into the simplified expression.

WORKED EXAMPLE

Find lim as x -> 11 of $(x^2 - 121)/(x - 11)$.

1. Factor the numerator: $x^2 - 121 = (x - 11)(x + 11)$.
2. For x not equal to 11, cancel x - 11 to get x + 11.
3. As x approaches 11, x + 11 approaches 22.

Answer: 22

PRACTICE 6 problems

1 Find lim as x -> 2 of $(x^2 - 4)/(x - 2)$.

ANSWER _____

2 Find lim as x -> 3 of $(x^2 + 2x - 15)/(x - 3)$.

ANSWER _____

3 Find lim as x -> -1 of $(x^2 + 3x + 2)/(x + 1)$.

ANSWER _____

4 Find lim as x -> 4 of (sqrt(x + 5) - 3)/(x - 4).

ANSWER _____

5 Find lim as x -> 0 of (1/(x + 5) - 1/5)/x.

ANSWER _____

6 Find lim as x -> -2 of $(x^3 + 8)/(x + 2)$.

ANSWER _____

APPLY IT Show your work

1 During a bike ride, your distance in meters after t seconds is $d(t) = t^2 + 4t$. Find the limit of $(d(t) - d(2))/(t - 2)$ as t approaches 2. What does the result tell you about your speed at that moment?

WORK SPACE

ANSWER _____

2 A phone's battery level, in percent, is modeled by $B(t) = 20 + 10\text{sqrt}(t + 1)$, where t is minutes since charging began. Find the limit of $(B(t) - B(3))/(t - 3)$ as t approaches 3. What does the result tell you about the battery level at that moment?

WORK SPACE

ANSWER _____

CHALLENGE One step up

Find lim as $x \to 2$ of $(x^2 - 4)/(\text{sqrt}(x + 2) - 2)$. Explain why canceling a factor is valid when finding this limit.

WORK SPACE AND ANSWER

Limits That Simplify

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find lim as x -> 2 of $(x^2 - 4)/(x - 2)$.	**4**
2	Find lim as x -> 3 of $(x^2 + 2x - 15)/(x - 3)$.	**8**
3	Find lim as x -> -1 of $(x^2 + 3x + 2)/(x + 1)$.	**1**
4	Find lim as x -> 4 of (sqrt(x + 5) - 3)/(x - 4).	**1/6**
5	Find lim as x -> 0 of (1/(x + 5) - 1/5)/x.	**-1/25**
6	Find lim as x -> -2 of $(x^3 + 8)/(x + 2)$.	**12**

PART 2 · APPLY IT

1. **8 meters per second, your speed at 2 seconds.** Since d(2) = 12, factor $(t^2 + 4t - 12)/(t - 2)$ as (t - 2)(t + 6)/(t - 2). The limit is 8 meters per second.
2. **5/2 percentage points per minute, the rate at which the battery level is rising at 3 minutes.** Since B(3) = 40, rationalize 10(sqrt(t + 1) - 2)/(t - 3) to get 10/(sqrt(t + 1) + 2). The limit is 5/2 percentage points per minute.

CHALLENGE

16. For x not equal to 2, rationalizing the denominator and canceling x - 2 gives (x + 2)(sqrt(x + 2) + 2), which approaches 16.

For the challenge, accept any valid simplification that shows the expression approaches 16. Canceling a factor at the excluded input is valid for finding a limit because the limit depends on values near that input, not at it.