LESSON MAKER PRO

Limits in Motion

Find where a limit goes, even when substitution fails.

HIGH SCHOOL · C.A.9

C.A.9 "Verify the behavior and direction of non-determinable limits."

NAME _____ DATE _____

LOOK BEYOND SUBSTITUTION

Start by substituting the value x approaches. If a factorable rational expression gives 0/0, cancel common factors and try again. If the denominator still approaches zero, check the signs on each side; for a ratio of equal-degree polynomials as x grows without bound, use the ratio of their leading coefficients.

WORKED EXAMPLE

Find the limit of $(x^2 - 49)/(x - 7)$ as x approaches 7.

1. Substituting 7 gives 0/0, so factor the numerator.
2. Rewrite the expression as $(x - 7)(x + 7)/(x - 7)$.
3. For x not equal to 7, cancel $x - 7$ to get $x + 7$.
4. As x approaches 7, $x + 7$ approaches 14.

Answer: 14

PRACTICE 6 problems

1 Find the limit of $(x^2 - 9)/(x - 3)$ as x approaches 3.

ANSWER _____

2 Find the limit of $(x^2 + 3x + 2)/(x + 2)$ as x approaches -2.

ANSWER _____

3 Find the limit of $1/(x - 2)$ as x approaches 2 from the left.

ANSWER _____

4 Find the limit of $1/(x - 2)$ as x approaches 2 from the right.

ANSWER _____

5 For $(x + 1)/x$ as x approaches 0, find the left-hand and right-hand limits. Does the two-sided limit exist?

ANSWER _____

6 Find the limit of $(3x^2 + 1)/(x^2 + 2)$ as x approaches +infinity.

ANSWER _____

APPLY IT Show your work

1 During a bike ride, your distance after t seconds is modeled by $s(t) = t^2 + 4t$ meters. Your average speed from $t = 2$ to $t = 2 + h$ is $(s(2 + h) - s(2))/h$. What speed does this approach as h approaches 0?

WORK SPACE

ANSWER _____

2 A video app models loading time by $T(r) = 60/(r - 5)$ seconds when connection speed r is greater than 5 MB/s. What happens to the modeled loading time as r approaches 5 from above?

WORK SPACE

ANSWER _____

CHALLENGE One step up

Find both one-sided limits of $(x^2 - 4)/(x - 2)^2$ as x approaches 2. Does the two-sided limit exist? Explain using the signs of the simplified expression.

WORK SPACE AND ANSWER

Limits in Motion

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the limit of $(x^2 - 9)/(x - 3)$ as x approaches 3.	**6**
2	Find the limit of $(x^2 + 3x + 2)/(x + 2)$ as x approaches -2.	**-1**
3	Find the limit of $1/(x - 2)$ as x approaches 2 from the left.	**-infinity**
4	Find the limit of $1/(x - 2)$ as x approaches 2 from the right.	**+infinity**
5	For $(x + 1)/x$ as x approaches 0, find the left-hand and right-hand limits. Does the two-sided limit exist?	**Left: -infinity. Right: +infinity. The two-sided limit does not exist.**
6	Find the limit of $(3x^2 + 1)/(x^2 + 2)$ as x approaches +infinity.	**3**

PART 2 · APPLY IT

1. **8 meters per second** Substitute $s(2 + h) = 12 + 8h + h^2$ and $s(2) = 12$. The average speed simplifies to $8 + h$, which approaches 8 meters per second.
2. **The modeled loading time increases without bound, approaching +infinity.** As r approaches 5 from above, r - 5 stays positive and approaches 0. Dividing 60 by an increasingly small positive number gives increasingly large loading times.

CHALLENGE

Cancel one factor to get (x + 2)/(x - 2), which approaches -infinity from the left and +infinity from the right. The two-sided limit does not exist because the one-sided limits differ.

Accept DNE for a two-sided limit that does not exist, but do not treat DNE as a direction. In the challenge, students must identify which side approaches each infinity.