LESSON MAKER PRO

Change in Motion

Compare change over an interval with change at one moment.

HIGH SCHOOL · C.G.22

C.G.22 "Demonstrate and explain the differences between average and instantaneous rates of change."

NAME _____ DATE _____

TWO KINDS OF RATE

Average rate of change is the change in output divided by the change in input over an interval. Instantaneous rate is the rate at one input value. For $f(x) = ax^2 + bx + c$, the instantaneous rate at $x = k$ is $2ak + b$.

WORKED EXAMPLE

For $s(t) = t^2$, find the average rate from $t = 7$ to $t = 8$ and the instantaneous rate at $t = 8$.

VALUES OF S(T)

t	s(t)
7	49
8	64

1. Average rate: $(64 - 49)/(8 - 7) = 15$.
2. Instantaneous rate: $((8 + h)^2 - 64)/h = 16 + h$.
3. As h approaches 0, $16 + h$ approaches 16.

Answer: Average rate: 15. Instantaneous rate: 16.

PRACTICE 4 problems

1 Find the average rate of change of $f(x) = 3x^2$ from $x = 0$ to $x = 2$.

ANSWER _____

2 Find the instantaneous rate of change of $f(x) = 2x^2 + x$ at $x = 3$.

ANSWER _____

3 For $f(x) = x^2 + 3x$, find the average rate from $x = 0$ to $x = 3$ and the instantaneous rate at $x = 3$.

ANSWER _____

4 For $f(x) = x^2$, find the average rate from $x = 0$ to $x = 3$ and the instantaneous rate at $x = 0$. Which is larger?

ANSWER _____

APPLY IT Show your work

1 A video's views follow $v(t) = 30t^2$, where t is hours since posting. Find the average rate from hour 1 to hour 3 and the instantaneous rate at hour 3.

VIDEO VIEWS

Hours	Views
1	30
3	270

ANSWER _____

2 A rider's distance is $d(t) = t^2 + 2t$ meters after t seconds. Find the average speed from $t = 0$ to $t = 2$ and the instantaneous speed at $t = 2$.

WORK SPACE

ANSWER _____

CHALLENGE One step up

For $f(x) = x^2 + 2x$, compare the average rates on [0, 2] and [1, 3] with the instantaneous rate at $x = 2$. Which average matches it? Explain why.

WORK SPACE AND ANSWER

Change in Motion

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the average rate of change of $f(x) = 3x^2$ from $x = 0$ to $x = 2$.	**6**
2	Find the instantaneous rate of change of $f(x) = 2x^2 + x$ at $x = 3$.	**13**
3	For $f(x) = x^2 + 3x$, find the average rate from $x = 0$ to $x = 3$ and the instantaneous rate at $x = 3$.	**Average rate: 6. Instantaneous rate: 9.**
4	For $f(x) = x^2$, find the average rate from $x = 0$ to $x = 3$ and the instantaneous rate at $x = 0$. Which is larger?	**Average rate: 3. Instantaneous rate: 0. The average rate is larger.**

PART 2 · APPLY IT

1. **Average: 120 views per hour. Instantaneous: 180 views per hour.** The average is (270 - 30)/(3 - 1) = 120. The instantaneous rate at t = 3 is 60(3) = 180.
2. **Average: 4 m/s. Instantaneous: 6 m/s.** The average is (d(2) - d(0))/2 = 8/2 = 4. The instantaneous speed at t = 2 is 2(2) + 2 = 6.

CHALLENGE

The average rates are 4 on [0, 2] and 6 on [1, 3]. The instantaneous rate at x = 2 is 6, so [1, 3] matches. For this quadratic, the average rate on [1, 3] equals the instantaneous rate at its midpoint, x = 2.

Accept equivalent rate calculations. In the challenge, accept an explanation using the symmetric endpoints 1 and 3 or the computed rates.