LESSON MAKER PRO

Find the Turns

Use derivatives to describe a curve.

HIGH SCHOOL · C.G.23

C.G.23 "Apply differentiation techniques to curve sketching"

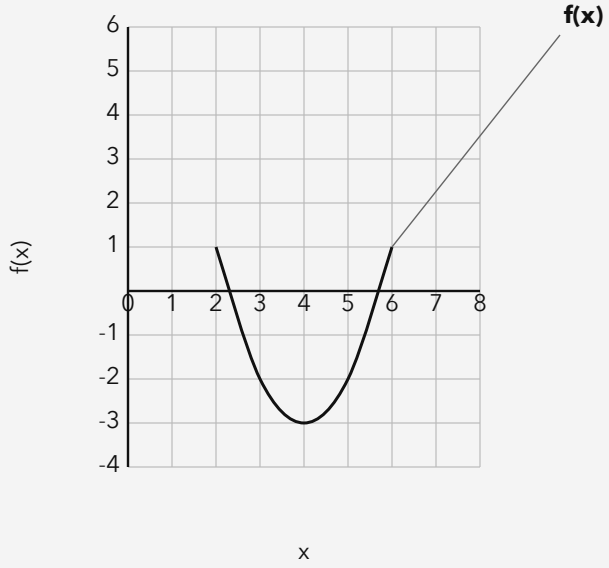
NAME _____ DATE _____

STATIONARY POINTS

Solve f'(x)=0 to find where a curve may turn. If f' changes from negative to positive, the point is a local minimum. If it changes from positive to negative, the point is a local maximum.

WORKED EXAMPLE

Find and classify the turning point of $f(x)=x^2-8x+13$.



1. f'(x)=2x-8. Set 2x-8=0 to get x=4.
2. f(4)=16-32+13=-3.
3. f' changes from negative to positive at x=4.

Answer: (4, -3), a local minimum

PRACTICE 4 problems

1 Find and classify the turning point of $f(x)=x^2-4x+7$.

ANSWER _____

2 Find and classify the turning point of $f(x)=-x^2+6x+1$.

ANSWER _____

3 For $f(x)=x^3-3x$, is the stationary point at $x=1$ a maximum or minimum? Give its y-value.

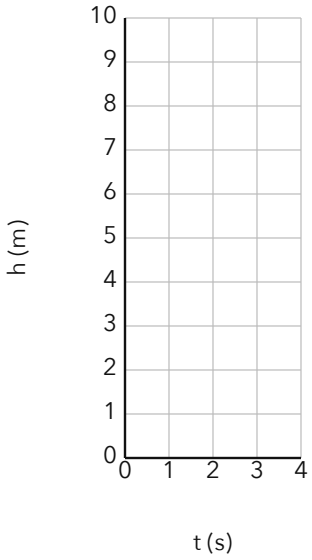
ANSWER _____

4 Find and classify both stationary points of $f(x)=x^3-12x$.

ANSWER _____

APPLY IT Show your work

1 A drone's height is $h(t)=-t^2+4t+5$ meters for $0\le t\le 4$ seconds. Sketch the curve. Mark the peak and give its time and height.



ANSWER _____

2 A club's profit after x one-dollar price increases is $P(x)=-x^2+10x+15$ dollars, where $0\le x\le 10$. How many increases give the greatest profit, and what is it?

WORK SPACE

ANSWER _____

CHALLENGE One step up

For $f(x)=x^3-6x^2+9x$ on $-1\le x\le 4$, find and classify both stationary points. Where does the curve rise and fall?

WORK SPACE AND ANSWER

Find the Turns

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find and classify the turning point of $f(x)=x^2-4x+7$.	**(2, 3), a minimum**
2	Find and classify the turning point of $f(x)=-x^2+6x+1$.	**(3, 10), a maximum**
3	For $f(x)=x^3-3x$, is the stationary point at $x=1$ a maximum or minimum? Give its y-value.	**Minimum; y=-2**
4	Find and classify both stationary points of $f(x)=x^3-12x$.	**(-2, 16), a local maximum; (2, -16), a local minimum**

PART 2 · APPLY IT

1. **Peak: (2, 9). The drone reaches 9 meters at 2 seconds. The sketch passes through (0, 5), (1, 8), (3, 8), and (4, 5).** Set $h'(t)=-2t+4$ to zero to get $t=2$. Then $h(2)=9$; plot the peak and the other listed points.
2. **5 increases; $40 profit** $P'(x)=-2x+10$ is zero at $x=5$ and changes from positive to negative. $P(5)=40$.

CHALLENGE

(1, 4) is a local maximum; (3, 0) is a local minimum. The curve rises on (-1, 1) and (3, 4), and falls on (1, 3).

Accept increasing for rising and decreasing for falling. Sketches need the correct peak and a downward-opening curve through the listed points.