LESSON MAKER PRO

Find the Tangent

Match tangent slopes to secant slopes.

HIGH SCHOOL · C.G.24

C.G.24 "Apply Rolle's Theorem and the Mean Value Theorem and their geometric consequences."

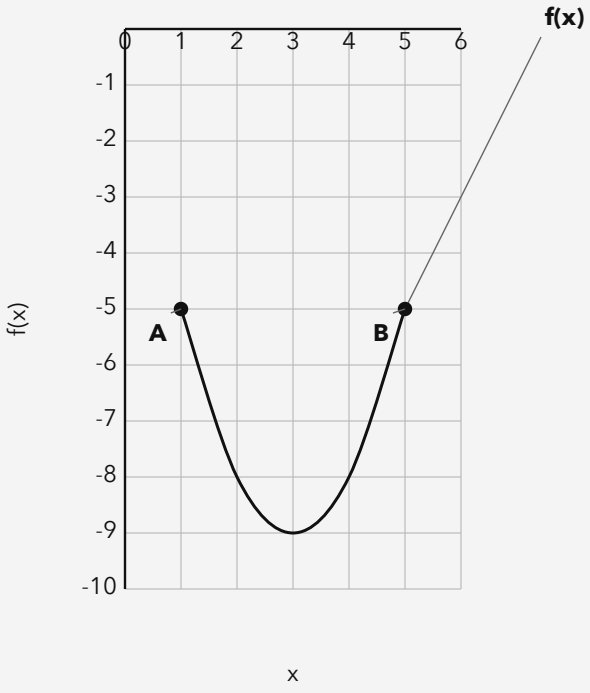
NAME _____ DATE _____

TWO THEOREMS, ONE SLOPE

If f is continuous on [a,b] and differentiable on (a,b), the Mean Value Theorem says some tangent slope equals the secant slope, (f(b)-f(a))/(b-a). If f(a)=f(b), Rolle's Theorem says some tangent is horizontal.

WORKED EXAMPLE

For $f(x)=x^2-6x$ on [1,5], find c using Rolle's Theorem.



1. f(1)=-5 and f(5)=-5, so Rolle's Theorem applies.
2. Find the derivative: f'(x)=2x-6.
3. Set f'(c)=0: 2c-6=0, so c=3.

Answer: c=3

PRACTICE 4 problems

1 Use the Mean Value Theorem to find c for $f(x)=x^2$ on [0,4].

ANSWER _____

2 Use Rolle's Theorem to find c for $f(x)=x^2-8x$ on [2,6].

ANSWER _____

3 Use the Mean Value Theorem to find c for $f(x)=x^3$ on [-1,2].

ANSWER _____

4 Find every c given by Rolle's Theorem for $f(x)=x^3-4x$ on [-2,2].

ANSWER _____

APPLY IT Show your work

1 An e-scooter travels $d(t)=t^2+2t$ meters in t seconds, for $0 \le t \le 4$. When does its speed equal its average speed?

WORK SPACE

ANSWER _____

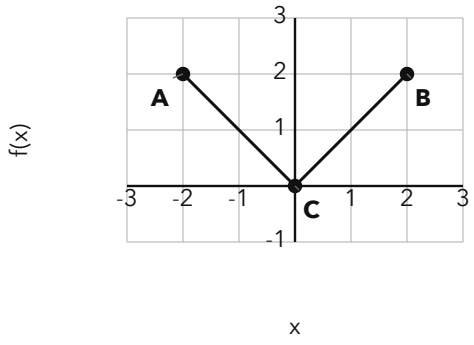
2 A drone's height is $h(t)=-t^2+4t+10$ meters, for $0 \le t \le 4$ seconds. Its starting and ending heights match. When is its vertical speed zero?

WORK SPACE

ANSWER _____

CHALLENGE One step up

This graph has f(-2)=f(2). Does Rolle's Theorem guarantee a horizontal tangent on (-2,2)? Explain.



WORK SPACE AND ANSWER

ANSWER KEY

Find the Tangent

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Use the Mean Value Theorem to find c for $f(x)=x^2$ on [0,4].	**c=2**
2	Use Rolle's Theorem to find c for $f(x)=x^2-8x$ on [2,6].	**c=4**
3	Use the Mean Value Theorem to find c for $f(x)=x^3$ on [-1,2].	**c=1**
4	Find every c given by Rolle's Theorem for $f(x)=x^3-4x$ on [-2,2].	**c=-2sqrt(3)/3 and c=2sqrt(3)/3**

PART 2 · APPLY IT

1. **t=2 seconds** The average speed is (d(4)-d(0))/4=6 m/s. Solve d'(t)=2t+2=6.
2. **t=2 seconds** The polynomial meets Rolle's Theorem's conditions, and h(0)=h(4)=10. Solve h'(t)=-2t+4=0.

CHALLENGE

No. The graph has a corner at x=0, so f is not differentiable throughout (-2,2). Neither segment has a horizontal tangent.

Accept equivalent exact forms for the two values in problem 4. In the challenge, the corner at x=0 is the missing condition.