LESSON MAKER PRO

Close Enough Lines

Use a tangent line to estimate nearby values.

HIGH SCHOOL · C.G.25

C.G.25 "Identify and apply local linear approximations."

NAME _____ DATE _____

THE LOCAL LINE

Near x = a, a curve is close to its tangent line. If the curve has value f(a) and slope m at a, use L(x) = f(a) + m(x - a). For f(x) = sqrt(x), choose a nearby perfect square $a = b^2$; then f(a) = b and m = 1/(2b).

WORKED EXAMPLE

Use a local linear approximation to estimate sqrt(48.3).

1. Choose a = 49, so b = 7 and f(49) = 7.
2. Find the slope: m = 1/(2 · 7) = 1/14.
3. Evaluate the line: L(48.3) = 7 + (48.3 - 49)/14 = 7 - 0.7/14 = 6.95.

Answer: L(48.3) = 6.95, so sqrt(48.3) is approximately 6.95.

PRACTICE 6 problems

1 For f(x) = sqrt(x), write L(x) at a = 4. Then use it to estimate sqrt(4.4).

ANSWER _____

2 Use a nearby perfect square to find a local linear estimate of sqrt(8.7).

ANSWER _____

3 Use a nearby perfect square to find a local linear estimate of sqrt(16.8).

ANSWER _____

4 Use a nearby perfect square to find a local linear estimate of sqrt(24).

ANSWER _____

5 For f(x) = sqrt(x), write L(x) at a = 36. Then use it to estimate sqrt(35.4).

ANSWER _____

6 Use a nearby perfect square to find a local linear estimate of sqrt(65.6).

ANSWER _____

APPLY IT Show your work

1 A square sticker has an area of 100.8 square centimeters. Use a local linear approximation near 100 to estimate its side length in centimeters.

WORK SPACE

ANSWER _____

2 A cyclist's distance from home is modeled by $d(t) = 3t^2$ kilometers, where t is hours. At t = 2, the distance graph has a slope of 12 kilometers per hour. Use a local linear approximation at t = 2 to estimate the distance from home at t = 2.1.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Estimate sqrt(80) using a local linear approximation at a = 64 and another at a = 81. Which estimate is closer to sqrt(80)? Justify your choice without a calculator.

WORK SPACE AND ANSWER

Close Enough Lines

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For f(x) = sqrt(x), write L(x) at a = 4. Then use it to estimate sqrt(4.4).	**L(x) = 2 + (x - 4)/4; L(4.4) = 2.1.**
2	Use a nearby perfect square to find a local linear estimate of sqrt(8.7).	**L(8.7) = 2.95, using a = 9.**
3	Use a nearby perfect square to find a local linear estimate of sqrt(16.8).	**L(16.8) = 4.1, using a = 16.**
4	Use a nearby perfect square to find a local linear estimate of sqrt(24).	**L(24) = 4.9, using a = 25.**
5	For f(x) = sqrt(x), write L(x) at a = 36. Then use it to estimate sqrt(35.4).	**L(x) = 6 + (x - 36)/12; L(35.4) = 5.95.**
6	Use a nearby perfect square to find a local linear estimate of sqrt(65.6).	**L(65.6) = 8.1, using a = 64.**

PART 2 · APPLY IT

1. **The estimated side length is 10.04 centimeters.** The side length is sqrt(100.8). At a = 100, the side length is 10 and the slope is 1/20, so L(100.8) = 10 + 0.8/20 = 10.04.
2. **The estimated distance from home is 13.2 kilometers.** At t = 2, d(2) = 12 kilometers. Use the given slope to get L(2.1) = 12 + 12(2.1 - 2) = 13.2.

CHALLENGE

At a = 64, the estimate is 8 + (80 - 64)/16 = 9. At a = 81, it is 9 + (80 - 81)/18 = 161/18. The estimate 161/18 is closer: both estimates exceed sqrt(80), but $9^2 - 80 = 1$ while $(161/18)^2 - 80 = 1/324$.

Accept equivalent fractions or decimals for the local linear estimates. These are exact values of the tangent-line models, not exact square roots.