LESSON MAKER PRO

Motion in Motion

Use velocity integrals to track change in position and distance traveled.

HIGH SCHOOL · C.S.28

C.S.28 "Utilize integrals to model and find solutions to real-world problems such as calculating displacement and total distance traveled."

NAME _____ DATE _____

SIGNED CHANGE OR TOTAL DISTANCE?

Integrate velocity v(t) over a time interval to find displacement, the signed change in position. To find total distance, split the interval wherever v(t)=0, then add the absolute values of the integrals over each part.

WORKED EXAMPLE

A cart has velocity v(t)=8-4t meters per second from t=0 to t=3 seconds. Find its displacement and total distance.

1. Find the direction change: 8-4t=0 when t=2.
2. Find an antiderivative: $F(t)=8t-2t^2$.
3. Displacement = F(3)-F(0)=6-0=6 m.
4. Total distance = |F(2)-F(0)|+|F(3)-F(2)|=8+2=10 m.

Answer: Displacement: 6 m; total distance: 10 m.

PRACTICE 6 problems

1 A cart's velocity is v(t)=2t+1 meters per second. Find its displacement from t=0 to t=2 seconds.

ANSWER _____

2 A cart's velocity is v(t)=t-5 meters per second. Find its displacement from t=0 to t=2 seconds.

ANSWER _____

3 A cart's velocity is v(t)=3-t meters per second. Find its displacement and total distance from t=0 to t=4 seconds.

ANSWER _____

4 A cart's velocity is v(t)=2t-6 meters per second. Find its total distance from t=0 to t=5 seconds.

ANSWER _____

5 A cart's velocity is $v(t)=t^2-1$ meters per second. Find its displacement and total distance from t=0 to t=2 seconds.

ANSWER _____

6 A cart's velocity is v(t)=-2t-1 meters per second. Find its displacement and total distance from t=1 to t=3 seconds.

ANSWER _____

APPLY IT Show your work

1 A skateboarder rolls along a straight path. Her velocity is v(t)=2t-10 meters per second from t=0 to t=7 seconds. Find her displacement and total distance.

WORK SPACE

ANSWER _____

2 In a video game, a character moves along a straight path with velocity v(t)=6-2t game units per second from t=0 to t=5 seconds. Find the character's displacement and total distance.

WORK SPACE

ANSWER _____

CHALLENGE One step up

A robot moves along a straight track with velocity v(t)=(t-1)(t-3) meters per second from t=0 to t=4 seconds. Find its displacement and total distance. Explain how you divided the time interval.

WORK SPACE AND ANSWER

Motion in Motion

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	A cart's velocity is v(t)=2t+1 meters per second. Find its displacement from t=0 to t=2 seconds.	**6 m**
2	A cart's velocity is v(t)=t-5 meters per second. Find its displacement from t=0 to t=2 seconds.	**-8 m**
3	A cart's velocity is v(t)=3-t meters per second. Find its displacement and total distance from t=0 to t=4 seconds.	**Displacement: 4 m; total distance: 5 m.**
4	A cart's velocity is v(t)=2t-6 meters per second. Find its total distance from t=0 to t=5 seconds.	**13 m**
5	A cart's velocity is v(t)=t^2-1 meters per second. Find its displacement and total distance from t=0 to t=2 seconds.	**Displacement: 2/3 m; total distance: 2 m.**
6	A cart's velocity is v(t)=-2t-1 meters per second. Find its displacement and total distance from t=1 to t=3 seconds.	**Displacement: -10 m; total distance: 10 m.**

PART 2 · APPLY IT

1. **Displacement: -21 m; total distance: 29 m.** An antiderivative is F(t)=t^2-10t, so displacement is F(7)-F(0)=-21 m. Velocity changes sign at t=5; the two distances are 25 m and 4 m.
2. **Displacement: 5 game units; total distance: 13 game units.** An antiderivative is F(t)=6t-t^2, so displacement is F(5)-F(0)=5 game units. Velocity changes sign at t=3; the two distances are 9 and 4 game units.

CHALLENGE

Displacement: 4/3 m; total distance: 4 m. Divide the interval at t=1 and t=3, where velocity is zero. The signed changes on [0,1], [1,3], and [3,4] are 4/3 m, -4/3 m, and 4/3 m; add their absolute values for total distance.

Displacement can be negative, but total distance cannot. Accept equivalent exact fractions and solutions that use geometric areas instead of antiderivatives where applicable.