LESSON MAKER PRO

# Sums Become Integrals

*Use equal subdivisions to find a limit.*

**HIGH SCHOOL · C.S.29**

**C.S.29** "Interpret the concept of definite integral as a limit of Riemann sums over equal subdivisions."

NAME _____ DATE _____

## THE BIG IDEA

Divide [a, b] into n equal parts. Each width is $\Delta x = (b - a)/n$, and the right endpoint of part i is $x_i = a + i\Delta x$. As $n \to \infty$, the sum of $f(x_i)\Delta x$ approaches the definite integral $\int$ from a to b f(x) dx.

## WORKED EXAMPLE

Write $\int$ from 0 to 5 $(3x + 1)\,dx$ as a limit of right-endpoint sums, then find its value.

1. $\Delta x = 5/n$ and $x_i = 5i/n$.
2. $S_n = (5/n)\,\Sigma$ from i = 1 to n of $(15i/n + 1)$.
3. Using $\Sigma$ from i = 1 to n of $i = n(n + 1)/2$, simplify: $S_n = 75(n + 1)/(2n) + 5$.
4. As $n \to \infty$, $S_n \to 75/2 + 5 = 85/2$.

**Answer: lim as n → ∞ of (5/n) Σ from i = 1 to n of (15i/n + 1) = 85/2**

## PRACTICE 6 problems

**1** Write $\int$ from 0 to 2 $(x + 1)\,dx$ as a limit of right-endpoint sums. Find its value.

**ANSWER** _____

**2** Find lim as $n \to \infty$ of $(1/n)\,\Sigma$ from i = 1 to n of $(i/n)^2$.

**ANSWER** _____

**3** Write $\int$ from 1 to 3 $2x\,dx$ as a limit of right-endpoint sums. Find its value.

**ANSWER** _____

**4** Find lim as $n \to \infty$ of $(4/n)\,\Sigma$ from i = 1 to n of $(4(i - 1)/n + 2)$.

**ANSWER** _____

**5** Write lim as $n \to \infty$ of $(3/n)\,\Sigma$ from i = 1 to n of $(1 + 3i/n)^2$ as a definite integral. Evaluate it.

**ANSWER** _____

**6** Write $\int$ from -1 to 1 $(x^2 + 1)\,dx$ as a limit of right-endpoint sums. Find its value.

**ANSWER** _____

## APPLY IT Show your work

**1** A game update downloads at a rate of $r(t) = 6 + 2t$ megabytes per minute, where t is minutes after the download starts. Write a limit of right-endpoint sums for the amount downloaded during the first 3 minutes. Find the amount.

WORK SPACE

**ANSWER** _____

**2** A portable charger supplies power at a rate of $p(t) = 12 - 2t$ watts, where t is hours after charging begins. Write a limit of left-endpoint sums for the energy supplied during the first 4 hours. Find the energy in watt-hours.

WORK SPACE

**ANSWER** _____

## CHALLENGE One step up

For $f(x) = x^2$ on [0, 2], show that the left- and right-endpoint sums have the same limit. Find that limit. You may use $\Sigma$ from $i = 1$ to $n$ of $i^2 = n(n + 1)(2n + 1)/6$.

WORK SPACE AND ANSWER

# Sums Become Integrals



## PART 1 · PRACTICE

| # | PROBLEM | ANSWER |
|---|---|---|
| 1 | Write ∫ from 0 to 2 (x + 1) dx as a limit of right-endpoint sums. Find its value. | **lim as n → ∞ of (2/n) Σ from i = 1 to n of (2i/n + 1) = 4** |
| 2 | Find lim as n → ∞ of (1/n) Σ from i = 1 to n of $(i/n)^2$. | **1/3** |
| 3 | Write ∫ from 1 to 3 2x dx as a limit of right-endpoint sums. Find its value. | **lim as n → ∞ of (2/n) Σ from i = 1 to n of (2 + 4i/n) = 8** |
| 4 | Find lim as n → ∞ of (4/n) Σ from i = 1 to n of (4(i - 1)/n + 2). | **16** |
| 5 | Write lim as n → ∞ of (3/n) Σ from i = 1 to n of $(1 + 3i/n)^2$ as a definite integral. Evaluate it. | **∫ from 1 to 4 $x^2$ dx = 21** |
| 6 | Write ∫ from -1 to 1 ($x^2$ + 1) dx as a limit of right-endpoint sums. Find its value. | **lim as n → ∞ of (2/n) Σ from i = 1 to n of $((-1 + 2i/n)^2 + 1) = 8/3$** |

## PART 2 · APPLY IT

1. **lim as n → ∞ of (3/n) Σ from i = 1 to n of (6 + 6i/n) = 27 megabytes** Each time interval has width 3/n, and its right endpoint is 3i/n. The limit is ∫ from 0 to 3 (6 + 2t) dt = 27 megabytes.
2. **lim as n → ∞ of (4/n) Σ from i = 1 to n of (12 - 8(i - 1)/n) = 32 watt-hours** Each time interval has width 4/n, and its left endpoint is 4(i - 1)/n. The limit is ∫ from 0 to 4 (12 - 2t) dt = 32 watt-hours.

## CHALLENGE

**The right sum is $R_n = (8/n^3)$ Σ from i = 1 to n of $i^2$, so $R_n \to 8/3$. The left sum is $L_n = (8/n^3)$ Σ from i = 1 to n of $(i - 1)^2$. Since $R_n - L_n = 8/n \to 0$, both sums have limit 8/3.**

Accept equivalent sum notation and simplified forms. In the challenge, accept any correct demonstration that both endpoint sums approach 8/3.