LESSON MAKER PRO

Factor or Prime?

Use discriminants and rational roots to justify your decisions.

HIGH SCHOOL · FA.A.34

FA.A.34 "Justify why some polynomials are prime over the rational number system."

NAME _____ DATE _____

HOW TO DECIDE

A quadratic with rational coefficients is prime over the rationals when its discriminant, $b^2 - 4ac$, is not the square of a rational number. For a cubic with integer coefficients, any rational root p/q in lowest terms has p dividing the constant term and q dividing the leading coefficient. A cubic is prime over the rationals if none of those candidates is a root.

WORKED EXAMPLE

Is $7x^2 + 11x + 9$ prime over the rationals? Justify.

1. Identify a = 7, b = 11, and c = 9.
2. Compute $b^2 - 4ac = 11^2 - 4(7)(9) = -131$.
3. Since -131 is not the square of a rational number, the quadratic has no rational roots.

Answer: $7x^2 + 11x + 9$ is prime over the rationals.

PRACTICE 6 problems

1 Is $x^2 + 4x + 7$ prime over the rationals? Use the discriminant to justify your answer.

ANSWER _____

2 Is $x^2 + 5x + 6$ prime over the rationals? Use the discriminant and, if it is not prime, give a factorization.

ANSWER _____

3 Is $2x^2 + 3x - 1$ prime over the rationals? Use the discriminant to justify your answer.

ANSWER _____

4 Is $3x^2 + 6x + 3$ prime over the rationals? Use the discriminant and, if it is not prime, give a factorization.

ANSWER _____

5 Is $x^3 + 2x + 1$ prime over the rationals? Check every possible rational root to justify your answer.

ANSWER _____

6 Is $x^3 + 3x^2 + 2x + 6$ prime over the rationals? Check a possible rational root and justify your answer.

ANSWER _____

APPLY IT Show your work

1 A sticker designer uses $A(x) = x^2 + 6x + 10$ to model a sticker's area, in square centimeters, for size setting x. Can A(x) be written as a product of two nonconstant polynomials with rational coefficients? Justify.

WORK SPACE

ANSWER _____

2 A game designer models a bonus after x completed levels with $B(x) = x^3 + x + 1$. Can B(x) be factored into two nonconstant polynomials with rational coefficients? Use every possible rational root to justify your answer.

WORK SPACE

ANSWER _____

CHALLENGE One step up

A student checks only $x = 1$ and $x = -1$, then concludes that $2x^3 + x + 1$ is prime over the rationals. Is that enough evidence? Check any missing rational-root candidates and finish the proof.

WORK SPACE AND ANSWER

Factor or Prime?

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Is $x^2 + 4x + 7$ prime over the rationals? Use the discriminant to justify your answer.	**Prime. The discriminant is $4^2 - 4(1)(7) = -12$, which is not a rational square.**
2	Is $x^2 + 5x + 6$ prime over the rationals? Use the discriminant and, if it is not prime, give a factorization.	**Not prime. The discriminant is $5^2 - 4(1)(6) = 1$, and $x^2 + 5x + 6 = (x + 2)(x + 3)$.**
3	Is $2x^2 + 3x - 1$ prime over the rationals? Use the discriminant to justify your answer.	**Prime. The discriminant is $3^2 - 4(2)(-1) = 17$, which is not a rational square.**
4	Is $3x^2 + 6x + 3$ prime over the rationals? Use the discriminant and, if it is not prime, give a factorization.	**Not prime. The discriminant is $6^2 - 4(3)(3) = 0$, and $3x^2 + 6x + 3 = 3(x + 1)(x + 1)$.**
5	Is $x^3 + 2x + 1$ prime over the rationals? Check every possible rational root to justify your answer.	**Prime. The only possible rational roots are 1 and -1; they give 4 and -2, respectively. Neither is a root.**
6	Is $x^3 + 3x^2 + 2x + 6$ prime over the rationals? Check a possible rational root and justify your answer.	**Not prime. Substituting $x = -3$ gives 0, so $x + 3$ is a factor. The polynomial equals $(x + 3)(x^2 + 2)$.**

PART 2 · APPLY IT

1. **No. A(x) is prime over the rationals because its discriminant is $6^2 - 4(1)(10) = -4$, which is not a rational square.** Compute the discriminant. A negative number cannot be the square of a rational number.
2. **No. B(x) is prime over the rationals: its only possible rational roots are 1 and -1, which give 3 and -1, respectively.** The rational root test gives only 1 and -1 as candidates. Neither is a root, so the cubic cannot factor over the rationals.

CHALLENGE

No. The student must also check 1/2 and -1/2. At $x = 1, -1, 1/2$, and $-1/2$, the polynomial gives 4, -2, 7/4, and 1/4, respectively. None is zero, so the cubic is prime over the rationals.

Accept any correct justification that rules out rational roots for a quadratic or cubic. For a polynomial that is not prime, accept any valid factorization into nonconstant polynomials with rational coefficients.