LESSON MAKER PRO

Space in Solids

Use slices and matching solids to reason about volume.

HIGH SCHOOL · G-GMD.1

G-GMD.1 "Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments."

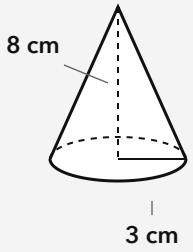
NAME _____ DATE _____

THE ONE-THIRD RULE

A cone or pyramid has one-third the volume of a matching cylinder or prism with the same base area and height. Imagine stacking thinner and thinner slices of a pointed solid. Their areas grow with the square of their distance from the tip, and their total approaches one-third of the matching straight solid's volume.

WORKED EXAMPLE

Find the volume of the cone shown. Give an exact answer.



1. Matching cylinder: V = pi × 3^2 × 8
2. Cylinder volume = 72pi cm^3
3. Cone volume = 72pi / 3 = 24pi cm^3

Answer: 24pi cm^3

PRACTICE 4 problems

1 Find the volume of a cone with radius 4 cm and height 9 cm.

ANSWER _____

2 A cylinder holds 150pi cm^3. What is the volume of a cone with the same base and height?

ANSWER _____

3 Find the volume of a cone with diameter 4 cm and height 15 cm.

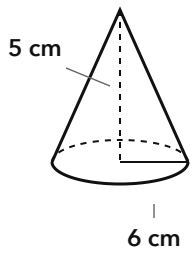
ANSWER _____

4 Find the volume of a pyramid with base area 24 cm^2 and height 9 cm.

ANSWER _____

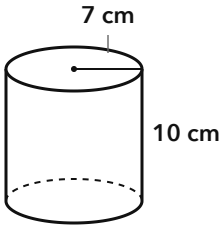
APPLY IT Show your work

1 A cone-shaped popcorn holder has the dimensions shown. How much space does it hold when filled to the rim? Give an exact volume.



ANSWER _____

2 A café fills the cylinder shown with lemonade. How many cone-shaped cups can it fill if each cup has the same base and height as the cylinder?



ANSWER _____

CHALLENGE One step up

Two cones have equal volumes. One has radius 4 cm and height 9 cm. The other has radius 6 cm. Find the second cone's height and explain your reasoning.

WORK SPACE AND ANSWER

ANSWER KEY

Space in Solids

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the volume of a cone with radius 4 cm and height 9 cm.	**48pi cm^3**
2	A cylinder holds 150pi cm^3. What is the volume of a cone with the same base and height?	**50pi cm^3**
3	Find the volume of a cone with diameter 4 cm and height 15 cm.	**20pi cm^3**
4	Find the volume of a pyramid with base area 24 cm^2 and height 9 cm.	**72 cm^3**

PART 2 · APPLY IT

1. **60pi cm^3** Use V = pi r^{2}h / 3. The volume is pi × 6^2 × 5 / 3 = 60pi cm^3.
2. **3 cups** Each cone-shaped cup holds one-third of the cylinder's volume. One full cylinder fills 3 cups.

CHALLENGE

4 cm. Equal cone volumes require equal values of r^{2}h, so 4^2 × 9 = 6^2 × h. Thus 144 = 36h and h = 4 cm.

Accept equivalent exact expressions using pi. For the challenge, accept any valid explanation that equates the cones' volumes.