Motion From Graphs

Derive motion rules from a velocity-time graph

PHY.1.4 "Use graphical analysis to derive kinematic equations."

NAME _____ DATE _____

1 READ AND MARK

Read it twice

YOUR JOB Circle the two graph features used to build a velocity rule. Underline how area gives displacement, and number the two shapes used to find that area.

A Cart in the Hallway

1. A motion sensor tracks a battery-powered cart as it rolls along a straight hallway. The table reports its velocity at four times after the cart passes a chalk mark. Imagine plotting each pair on a velocity-time graph, with time on the horizontal axis and velocity on the vertical axis. The four points form a straight line.

CART VELOCITY DATA

Time (s)	Velocity (m/s)
0	3
2	7
4	11
6	15

2. On a velocity-time graph, the vertical intercept gives the velocity at time zero. The line's slope is the change in velocity divided by the change in time, which is acceleration. Choose any two points on the straight line to find that slope. To build a velocity rule, start with the intercept and add slope multiplied by elapsed time.

3. Area under a velocity-time graph gives displacement. From time zero to time t, split the region below a rising line into a rectangle and a triangle. The rectangle has height equal to initial velocity; the triangle's height is the increase in velocity. For a shorter interval, use a trapezoid with endpoint velocities as parallel sides.

2 ANSWER WITH EVIDENCE

Use the passage

1. Use the points at 0 s and 4 s to find the slope of the cart's velocity-time line. What does that slope mean?

2. Use the line's intercept and slope to derive a rule for the cart's velocity v at time t.

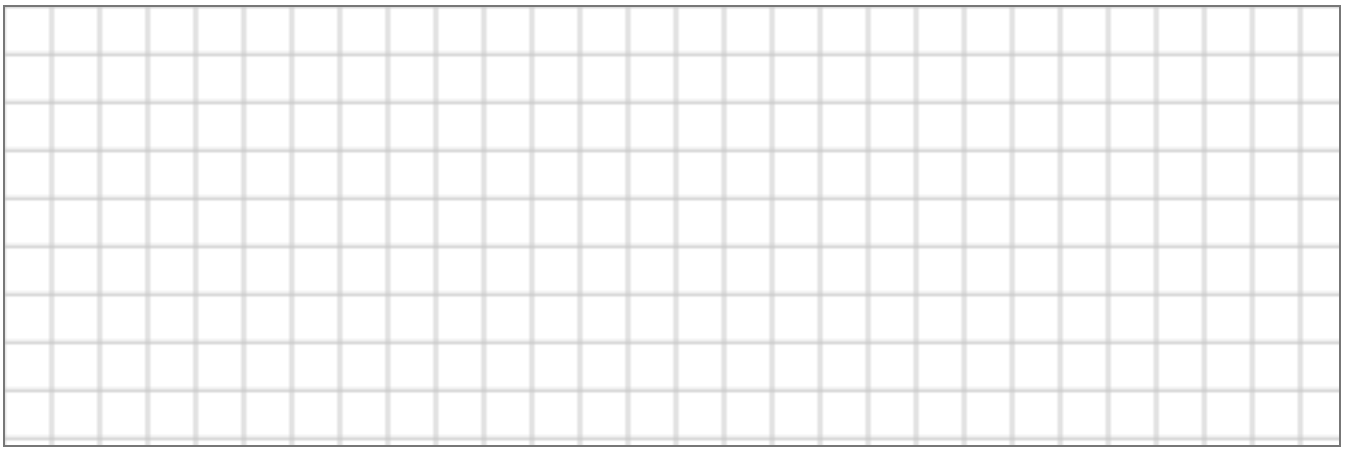
3. What displacement does the area under the velocity-time line give from 2 s to 6 s? Show how a trapezoid gives your answer.

4. Derive a rule for the cart's displacement Δx from 0 to t. Show the rectangle and triangle areas you add.

3 YOUR TURN

Your own words

On the grid, plot a different cart's velocity at (0 s, 5 m/s), (2 s, 8 m/s), and (4 s, 11 m/s), then draw its straight velocity-time line. Write two sentences that derive its velocity rule from slope and intercept and its displacement rule from area under the line.



Motion From Graphs

PART 1 · READ AND MARK

Circle the two graph features used to build a velocity rule. Underline how area gives displacement, and number the two shapes used to find that area. Accept any marking that follows this job and points at the right text.

PART 2 · ANSWERS

#	QUESTION	ANSWER
1	Use the points at 0 s and 4 s to find the slope of the cart's velocity-time line. What does that slope mean?	**(11 − 3) ÷ (4 − 0) = 2 m/s^2. The cart accelerates at 2 m/s^2.**
2	Use the line's intercept and slope to derive a rule for the cart's velocity v at time t.	**The intercept is 3 m/s and the slope is 2 m/s^2, so v(t) = 3 + 2t in m/s when t is in seconds.**
3	What displacement does the area under the velocity-time line give from 2 s to 6 s? Show how a trapezoid gives your answer.	**The endpoint velocities are 7 and 15 m/s. Trapezoid area = (7 + 15) ÷ 2 × 4 = 44 m.**
4	Derive a rule for the cart's displacement Δx from 0 to t. Show the rectangle and triangle areas you add.	**Rectangle area = 3t; triangle area = (1/2)(t)(2t) = t^2. Thus Δx(t) = 3t + t^2 in meters.**

PART 3 · YOUR TURN (SAMPLE ANSWER)

The plotted points (0, 5), (2, 8), and (4, 11) lie on a line with intercept 5 m/s and slope 1.5 m/s^2, giving v(t) = 5 + 1.5t in m/s. The area beneath this line from 0 to t is a rectangle plus a triangle, giving Δx(t) = 5t + 0.75t^2 in meters.

Accept equivalent equations with units stated separately. For Part 3, check that time is on the horizontal axis, velocity is on the vertical axis, and the line passes through all three points.