LESSON MAKER PRO

Forms Reveal Features

Rewrite quadratics to make zeros and maximum or minimum values visible.

HIGH SCHOOL · MT.CORE.ALG.FUN.7

MT.CORE.ALG.FUN.7 "Understand that expressions can be rewritten in equivalent forms to make different characteristics or features visible."

NAME _____ DATE _____

CHOOSE A USEFUL FORM

Factored form makes a quadratic's zeros visible: a product is zero when one of its factors is zero. Completing the square makes its maximum or minimum visible. A square cannot be negative, so adding a square gives a minimum, while subtracting a square gives a maximum.

WORKED EXAMPLE

Rewrite $x^2 + 18x + 65$ to show its minimum value.

1. Half of 18 is 9, and $9^2 = 81$.
2. $x^2 + 18x + 65 = (x^2 + 18x + 81) - 81 + 65$
3. $= (x + 9)^2 - 16$
4. The square is zero when $x = -9$.

Answer: $(x + 9)^2 - 16$; the minimum value is -16 when $x = -9$.

PRACTICE 6 problems

1 Rewrite $x^2 + 7x + 12$ in factored form.

ANSWER _____

2 Rewrite $x^2 - 3x - 10$ in factored form. Then find its zeros.

ANSWER _____

3 Rewrite $-x^2 + 2x + 15$ in factored form. Then find its zeros.

ANSWER _____

4 Complete the square to rewrite $x^2 - 8x + 11$ in a form that shows its minimum.

ANSWER _____

5 Rewrite $x^2 + 4x - 1$ by completing the square. State its minimum value and the x-value where it occurs.

ANSWER _____

6 Rewrite $-x^2 - 6x + 1$ by completing the square. State its maximum value and the x-value where it occurs.

ANSWER _____

APPLY IT Show your work

1 A drone's height in meters, x seconds after takeoff, is modeled by $H(x) = -x^2 + 8x + 3$ for $0 \le x \le 8$. Rewrite the expression to show the drone's maximum height. What is that height, and when does it occur?

WORK SPACE

ANSWER _____

2 A school club models its profit in dollars from sticker packs by $P(x) = -10x^2 + 110x - 180$, where x is the price of one pack in dollars and $2 \le x \le 9$. Rewrite P(x) in factored form. At which prices does the club break even?

WORK SPACE

ANSWER _____

CHALLENGE One step up

Rewrite $-2x^2 + 12x - 10$ in both factored form and a form that shows its maximum. Use the two forms to find its zeros and maximum value.

WORK SPACE AND ANSWER

ANSWER KEY

Forms Reveal Features

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Rewrite $x^2 + 7x + 12$ in factored form.	**$(x + 3)(x + 4)$**
2	Rewrite $x^2 - 3x - 10$ in factored form. Then find its zeros.	**$(x - 5)(x + 2)$; zeros: 5 and -2**
3	Rewrite $-x^2 + 2x + 15$ in factored form. Then find its zeros.	**$-(x - 5)(x + 3)$; zeros: 5 and -3**
4	Complete the square to rewrite $x^2 - 8x + 11$ in a form that shows its minimum.	**$(x - 4)^2 - 5$**
5	Rewrite $x^2 + 4x - 1$ by completing the square. State its minimum value and the x-value where it occurs.	**$(x + 2)^2 - 5$; minimum value: -5 when x = -2**
6	Rewrite $-x^2 - 6x + 1$ by completing the square. State its maximum value and the x-value where it occurs.	**$-(x + 3)^2 + 10$; maximum value: 10 when x = -3**

PART 2 · APPLY IT

1. **$H(x) = -(x - 4)^2 + 19$; maximum height: 19 meters after 4 seconds** Complete the square to get $-(x - 4)^2 + 19$. The subtracted square is zero at $x = 4$, so the height is greatest then.
2. **$P(x) = -10(x - 2)(x - 9)$; the club breaks even at $2 and $9 per pack** Factor out -10, then factor $x^2 - 11x + 18$ as $(x - 2)(x - 9)$. Profit is zero when either factor is zero.

CHALLENGE

Factored form: $-2(x - 1)(x - 5)$. Maximum form: $-2(x - 3)^2 + 8$. The zeros are 1 and 5, and the maximum value is 8 when x = 3.

Accept factors in either order and equivalent expressions with a negative sign placed in a different factor. For maximum and minimum questions, students must identify the value and the x-value where it occurs.