LESSON MAKER PRO

Fit the Trend

Use three data points to study a linear association.

HIGH SCHOOL · MT.CORE.DATA.INT.6

MT.CORE.DATA.INT.6 "Analyze the relationship between two quantitative data distributions in context that have a linear association. This standard should incorporate cultural context relating to Montana Indigenous Peoples and local communities by: Using..."

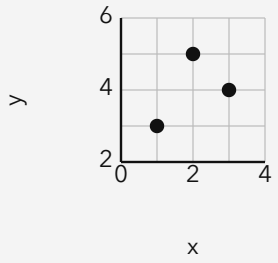
NAME _____ DATE _____

THE BIG IDEA

For three equally spaced x-values, the least-squares slope is m = (last y - first y)/(last x - first x). Find b = mean y - m(mean x), then write the line as y = mx + b. This line minimizes the sum of squared residuals, where a residual is observed y minus predicted y. Correlation measures linear association: r = Sxy/sqrt(Sxx Syy), where Sxy is the sum of products of distances from the means, and Sxx and Syy are the sums of squared distances from each mean.

WORKED EXAMPLE

Find the least-squares line and correlation r for the plotted points.



1. Mean x = 2 and mean y = 4.
2. m = (4 - 3)/(3 - 1) = 1/2.
3. b = 4 - (1/2)(2) = 3.
4. Sxy = 1, Sxx = 2, Syy = 2, so r = 1/sqrt(2 × 2) = 1/2.

Answer: y = (1/2)x + 3; r = 1/2

PRACTICE 4 problems

1 Find the least-squares line for (0, 2), (1, 3), (2, 4).

ANSWER _____

2 Find the least-squares line for (0, 5), (1, 4), (2, 3).

ANSWER _____

3 For (0, 2), (1, 4), (2, 4), use y = x + 1. Find the residual at x = 1 and its square.

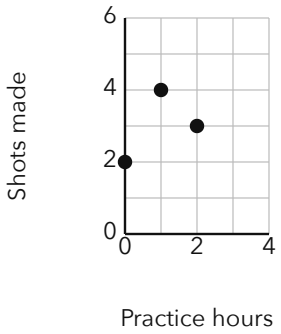
ANSWER _____

4 Find r for (0, 4), (1, 2), (2, 3).

ANSWER _____

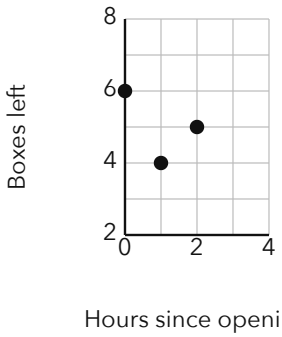
APPLY IT Show your work

1 A fictional Blackfeet youth basketball clinic tracks practice hours and shots made out of 10. Use the plot to find the least-squares line. Describe the association.



ANSWER _____

2 At a fictional Confederated Salish and Kootenai Tribes youth swap, the plot shows hours since opening and supply boxes left. Find r and describe the association.



ANSWER _____

CHALLENGE One step up

For (0, 1), (1, 3), (2, 2), compare the sums of squared residuals for y = (1/2)x + 3/2 and y = x + 1. Which line fits better, and why?

WORK SPACE AND ANSWER

ANSWER KEY

Fit the Trend

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the least-squares line for (0, 2), (1, 3), (2, 4).	**y = x + 2**
2	Find the least-squares line for (0, 5), (1, 4), (2, 3).	**y = -x + 5**
3	For (0, 2), (1, 4), (2, 4), use y = x + 1. Find the residual at x = 1 and its square.	**Residual = 4 - 2 = 2; squared residual = 4.**
4	Find r for (0, 4), (1, 2), (2, 3).	**r = -1/2**

PART 2 · APPLY IT

1. **y = (1/2)x + 5/2. The data show a positive linear association: more practice hours tend to go with more shots made.** The endpoint slope is (3 - 2)/(2 - 0) = 1/2. Both means are 1 and 3, respectively, so b = 3 - (1/2)(1) = 5/2.
2. **r = -1/2. The data show a negative linear association: later hours tend to go with fewer boxes left.** Mean x = 1 and mean y = 5. Sxy = -1, Sxx = 2, and Syy = 2, so r = -1/sqrt(2 × 2) = -1/2.

CHALLENGE

The sums are 3/2 and 2, respectively. y = (1/2)x + 3/2 fits better because its sum of squared residuals is smaller.

Accept equivalent equation forms and descriptions of positive or negative trends that do not claim causation.