LESSON MAKER PRO

# Why Base e?

*Find the limit of growth with ever more frequent updates.*

**HIGH SCHOOL · MT.PLUS.ALG.EXP.2**

**MT.PLUS.ALG.EXP.2** "Understand why e is defined as the natural base."

NAME _____ DATE _____

## THE NATURAL BASE

As n grows without bound, $(1+1/n)^n$ approaches e. For fixed numbers a and b, $(1+a/n)^{bn}$ approaches $e^{ab}$. This is why e appears when growth is divided into more and more frequent updates.

## WORKED EXAMPLE

As n grows without bound, what does $(1+7/n)^{8n}$ approach?

1. Rewrite the expression as $[(1+7/n)^{n/7}]^{56}$.
2. As n grows without bound, $(1+7/n)^{n/7}$ approaches e.
3. Raise e to the 56th power.

**Answer: $e^{56}$**

## PRACTICE 6 problems

**1** As n grows without bound, what does $(1+1/n)^n$ approach?

**ANSWER** _____

**2** As n grows without bound, what does $(1+1/n)^{6n}$ approach?

**ANSWER** _____

**3** As n grows without bound, what does $(1+2/n)^{5n}$ approach?

**ANSWER** _____

**4** As n grows without bound, what does $(1+5/n)^n$ approach?

**ANSWER** _____

**5** As n grows without bound, what does $(1-1/n)^{2n}$ approach?

**ANSWER** _____

**6** Find x so that $(1+x/n)^{2n}$ approaches $e^6$ as n grows without bound.

**ANSWER** _____

## APPLY IT Show your work

**1** You deposit $600 in a savings account that earns 5% interest per year, compounded n times per year. What is the exact balance after 2 years as n grows without bound?

WORK SPACE

**ANSWER** _____

**2** A creator's video starts with 200 views. A model predicts 50% growth per day, applied in n equal updates each day. What does the model predict after 6 days as n grows without bound? Give an exact expression.

WORK SPACE

**ANSWER** _____

## CHALLENGE One step up

As n grows without bound, do $(1+6/n)^{5n}$ and $(1+5/n)^{6n}$ approach the same value? Explain using the rule for e.

WORK SPACE AND ANSWER

# Why Base e?



## PART 1 · PRACTICE

| # | PROBLEM | ANSWER |
|---|---|---|
| 1 | As n grows without bound, what does $(1+1/n)^n$ approach? | **$e$** |
| 2 | As n grows without bound, what does $(1+1/n)^{6n}$ approach? | **$e^6$** |
| 3 | As n grows without bound, what does $(1+2/n)^{5n}$ approach? | **$e^{10}$** |
| 4 | As n grows without bound, what does $(1+5/n)^n$ approach? | **$e^5$** |
| 5 | As n grows without bound, what does $(1-1/n)^{2n}$ approach? | **$e^{-2}$** |
| 6 | Find x so that $(1+x/n)^{2n}$ approaches $e^6$ as n grows without bound. | **$x = 3$** |

## PART 2 · APPLY IT

1. **$\$600e^{1/10}$** The balance is $\$600(1+1/(20n))^{2n}$. The exponent on e is (1/20)(2) = 1/10.
2. **$200e^3$ views** The model gives $200(1+1/(2n))^{6n}$. The exponent on e is (1/2)(6) = 3.

## CHALLENGE

**Yes. The first approaches $e^{6(5)}$ and the second approaches $e^{5(6)}$, so both approach $e^{30}$.**

Accept equivalent exact expressions that use e. In the word problems, accept answers with or without units if the expressions are correct.