LESSON MAKER PRO

Two Forms, One Model

Switch between exponential and continuous-rate forms.

HIGH SCHOOL · MT.PLUS.ALG.EXP.3.a

MT.PLUS.ALG.EXP.3.a "Understand that exponential and logarithmic functions can be represented using multiple forms by: Expressing exponential functions in the form f(x) = a · b^x and f(x) = Pe^(r·t), and"

NAME _____ DATE _____

THE CONVERSION RULE

For $b > 0$, $b^t = e^{(\ln b)t}$. This means $a \cdot b^t$ and Pe^{rt} describe the same function when $P = a$ and $r = \ln b$.

WORKED EXAMPLE

Rewrite $h(t) = 11 \cdot (7/6)^t$ in the form $h(t) = Pe^{rt}$. State P and r.

1. Identify the starting value: $P = 11$.
2. Use the base to find the rate: $r = \ln(7/6)$.
3. Substitute into $h(t) = Pe^{rt}$: $h(t) = 11e^{(\ln(7/6))t}$.

Answer: $h(t) = 11e^{(\ln(7/6))t}$; $P = 11$, $r = \ln(7/6)$.

PRACTICE 6 problems

1 Rewrite $f(x) = 12 \cdot 2^x$ in the form $f(x) = Pe^{rx}$.

ANSWER _____

2 Rewrite $g(t) = 9 \cdot (1/2)^t$ in the form $g(t) = Pe^{rt}$.

ANSWER _____

3 Rewrite $h(t) = 20e^{(\ln 3)t}$ in the form $h(t) = a \cdot b^t$.

ANSWER _____

4 Rewrite $q(t) = 25e^{(\ln(1/4))t}$ in the form $q(t) = a \cdot b^t$.

ANSWER _____

5 A model starts at 16 and has a growth factor of 4 for each increase of 1 in t. Write the model in both $a \cdot b^t$ and Pe^{rt} forms.

ANSWER _____

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6 Rewrite $f(x) = 30e^{(-\ln 2)x}$ in the form $f(x) = a \cdot b^x$.

ANSWER _____

APPLY IT Show your work

1 A student-run music channel has 80 subscribers. Its subscriber count grows by 25% each month. Let t be months from now. Write the model in both $N(t) = a \cdot b^t$ and $N(t) = Pe^{rt}$ forms. State r.

WORK SPACE

ANSWER _____

2 A phone worth $480 loses 25% of its value each year. Let t be years since purchase. Write its value in both $V(t) = a \cdot b^t$ and $V(t) = Pe^{rt}$ forms. State whether r is positive or negative.

WORK SPACE

ANSWER _____

CHALLENGE One step up

An exponential model $h(t) = a \cdot b^t$, where $b > 0$, has the values in the table. Write h(t) in both $a \cdot b^t$ and Pe^{rt} forms. Explain how you found b.

MODEL VALUES

t	h(t)
0	18
2	8

WORK SPACE AND ANSWER

Two Forms, One Model

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Rewrite $f(x) = 12 \cdot 2^x$ in the form $f(x) = Pe^{rx}$.	**$f(x) = 12e^{(\ln 2)x}$**
2	Rewrite $g(t) = 9 \cdot (1/2)^t$ in the form $g(t) = Pe^{rt}$.	**$g(t) = 9e^{(\ln(1/2))t}$**
3	Rewrite $h(t) = 20e^{(\ln 3)t}$ in the form $h(t) = a \cdot b^t$.	**$h(t) = 20 \cdot 3^t$**
4	Rewrite $q(t) = 25e^{(\ln(1/4))t}$ in the form $q(t) = a \cdot b^t$.	**$q(t) = 25 \cdot (1/4)^t$**
5	A model starts at 16 and has a growth factor of 4 for each increase of 1 in t. Write the model in both $a \cdot b^t$ and Pe^{rt} forms.	**$f(t) = 16 \cdot 4^t$; $f(t) = 16e^{(\ln 4)t}$**
6	Rewrite $f(x) = 30e^{(-\ln 2)x}$ in the form $f(x) = a \cdot b^x$.	**$f(x) = 30 \cdot (1/2)^x$**

PART 2 · APPLY IT

1. **$N(t) = 80 \cdot (5/4)^t$; $N(t) = 80e^{(\ln(5/4))t}$; r = ln(5/4).** A 25% increase makes the monthly growth factor 1 + 1/4 = 5/4. Use the starting count for P and take the natural logarithm of the growth factor for r.
2. **$V(t) = 480 \cdot (3/4)^t$; $V(t) = 480e^{(\ln(3/4))t}$; r is negative.** The phone keeps 1 − 1/4 = 3/4 of its value each year. Since 3/4 is less than 1, ln(3/4) is negative.

CHALLENGE

$h(t) = 18 \cdot (2/3)^t$; $h(t) = 18e^{(\ln(2/3))t}$. Since h(0) = 18, a = 18. Then $18b^2 = 8$, so $b^2 = 4/9$ and the positive base is b = 2/3.

Accept r = −ln 2 in place of ln(1/2), and equivalent logarithmic expressions elsewhere. The continuous-rate form needs an exact rate, not a decimal approximation.