LESSON MAKER PRO

Log Lens

Write logarithms in base 10 and base e.

HIGH SCHOOL · MT.PLUS.ALG.EXP.3.b

MT.PLUS.ALG.EXP.3.b "Understand that exponential and logarithmic functions can be represented using multiple forms by: Expressing logarithmic functions in base 10 and base e."

NAME _____ DATE _____

CHANGE THE BASE

You can write a logarithm in another base by dividing two logarithms. For $b > 0$, $b \neq 1$, and $x > 0$, $\log_b(x) = \log(x)/\log(b) = \ln(x)/\ln(b)$. Here, log means base 10 and ln means base e.

WORKED EXAMPLE

Write $\log_{11}(121)$ using base 10 and base e, then find its exact value.

1. Base 10: $\log_{11}(121) = \log(121)/\log(11)$.
2. Base e: $\log_{11}(121) = \ln(121)/\ln(11)$.
3. Since $11^2 = 121$, both quotients equal 2.

Answer: log(121)/log(11) = ln(121)/ln(11) = 2

PRACTICE 6 problems

1 Write $\log_2(x)$ using base 10 and base e.

ANSWER _____

2 Write $\log_3(81)$ as a base-10 quotient. Then find its exact value.

ANSWER _____

3 Write $\log_5(125)$ as a base-e quotient. Then find its exact value.

ANSWER _____

4 Write log(16)/log(4) as one logarithm with a base. Then find its exact value.

ANSWER _____

5 Write ln(64)/ln(8) as one logarithm with a base. Then find its exact value.

ANSWER _____

6 Write $\log_9(27)$ as both a base-10 quotient and a base-e quotient. Then find its exact value.

ANSWER _____

APPLY IT Show your work

1 A video has 200 views. Its view count triples each day. How many days will it take to reach 5,400 views? Write the time using base-10 logs and using natural logs, then find the exact number of days.

WORK SPACE

ANSWER _____

2 During a long video call, a phone's battery charge halves each hour. It starts at 100%. How many hours will it take to reach 6.25%? Write the time using base-10 logs and using natural logs, then find the exact number of hours.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Explain why log(72)/log(6) and ln(72)/ln(6) are equal, but neither equals 2. Write their shared value as 2 plus a natural-log quotient.

WORK SPACE AND ANSWER

Log Lens

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Write $\log_2(x)$ using base 10 and base e.	**log(x)/log(2) and ln(x)/ln(2)**
2	Write $\log_3(81)$ as a base-10 quotient. Then find its exact value.	**log(81)/log(3) = 4**
3	Write $\log_5(125)$ as a base-e quotient. Then find its exact value.	**ln(125)/ln(5) = 3**
4	Write log(16)/log(4) as one logarithm with a base. Then find its exact value.	**$\log_4(16) = 2$**
5	Write ln(64)/ln(8) as one logarithm with a base. Then find its exact value.	**$\log_8(64) = 2$**
6	Write $\log_9(27)$ as both a base-10 quotient and a base-e quotient. Then find its exact value.	**log(27)/log(9) = ln(27)/ln(9) = 3/2**

PART 2 · APPLY IT

1. **log(27)/log(3) = ln(27)/ln(3) = 3 days** Divide 5,400 by 200 to get 27. Solve $3^t = 27$ using change of base; since $3^3 = 27$, t = 3.
2. **log(1/16)/log(1/2) = ln(1/16)/ln(1/2) = 4 hours** The charge falls to 1/16 of its starting level. Solve $(1/2)^t = 1/16$; since $(1/2)^4 = 1/16$, t = 4.

CHALLENGE

Both quotients equal $\log_6(72)$. Since $72 = 6^2 \cdot 2$, their shared value is 2 + ln(2)/ln(6), which is greater than 2.

Accept equivalent exact logarithmic expressions. Do not require decimal approximations.