LESSON MAKER PRO

Balanced Equation Paths

Solve equations by showing why each step preserves equality.

HIGH SCHOOL · AI-A.REI.1.a

AI-A.REI.1.a "Explain each step when solving a linear or quadratic equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution..."

NAME _____ DATE _____

KEEP BOTH SIDES EQUAL

Start by assuming the original equation is true. Each new equation must come from doing the same operation to both sides, such as adding the same number or dividing both sides by the same nonzero number. For a squared expression, include both values that can produce the same square.

WORKED EXAMPLE

Solve $11x - 13 = 31$. Show why each step keeps the equation balanced.

1. $11x - 13 = 31$
2. $11x = 44$, add 13 to both sides.
3. $x = 4$, divide both sides by 11.
4. Check: $11(4) - 13 = 31$.

Answer: x = 4

PRACTICE 6 problems

1 Solve $6x + 9 = 45$. Show two equation lines. Beside each line, state what you did to both sides.

ANSWER _____

2 Solve $5(x - 2) = 30$. State why dividing by 5 is allowed.

ANSWER _____

3 Solve $x/3 + 7 = 12$. Label the operation used in each step.

ANSWER _____

4 Solve $x^2 = 49$. Explain why there are two solutions.

ANSWER _____

5 Solve $(x + 1)(x - 8) = 0$. State the property that justifies your solution.

ANSWER _____

6 Solve $2x^2 - 18 = 0$. Show your equation steps, then explain the final step.

ANSWER _____

APPLY IT Show your work

1 The robotics club pays a $6 delivery fee plus $9 for each team T-shirt. The total cost is $60. Write and solve an equation for x, the number of T-shirts. Show why each operation keeps the equation balanced.

WORK SPACE

ANSWER _____

2 A soccer ball's height above the ground is modeled by $h = -2t^2 + 12t$, where t is time in seconds. When does the ball return to the ground after it is kicked? Set h equal to 0 and justify your solution method.

WORK SPACE

ANSWER _____

CHALLENGE One step up

A student writes: "$(x - 3)^2 = 25$, so $x - 3 = 5$, so $x = 8$." Is this argument complete? Write a corrected chain of equalities or cases, and explain the missing solution.

WORK SPACE AND ANSWER

ANSWER KEY

Balanced Equation Paths

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Solve $6x + 9 = 45$. Show two equation lines. Beside each line, state what you did to both sides.	**$6x = 36$, subtract 9 from both sides; $x = 6$, divide both sides by 6.**
2	Solve $5(x - 2) = 30$. State why dividing by 5 is allowed.	**$x - 2 = 6$, divide both sides by 5; $x = 8$, add 2 to both sides. Dividing both sides by the same nonzero number preserves equality.**
3	Solve $x/3 + 7 = 12$. Label the operation used in each step.	**$x/3 = 5$, subtract 7 from both sides; $x = 15$, multiply both sides by 3.**
4	Solve $x^2 = 49$. Explain why there are two solutions.	**$x = 7$ or $x = -7$. Since $x^2 = 49$, $x = \pm\sqrt{49}$.**
5	Solve $(x + 1)(x - 8) = 0$. State the property that justifies your solution.	**$x = -1$ or $x = 8$. By the Zero Product Property, if a product is 0, at least one factor is 0.**
6	Solve $2x^2 - 18 = 0$. Show your equation steps, then explain the final step.	**$2x^2 = 18$, then $x^2 = 9$; $x = 3$ or $x = -3$ because $x^2 = 9$ implies $x = \pm\sqrt{9}$.**

PART 2 · APPLY IT

1. **x = 6 T-shirts** Use $9x + 6 = 60$. Subtract 6 from both sides, then divide both sides by 9 to get $x = 6$.
2. **The ball returns to the ground 6 seconds after it is kicked.** Set $-2t^2 + 12t = 0$ and factor to get $-2t(t - 6) = 0$. The zeros are $t = 0$ and $t = 6$, and $t = 6$ is the time after the kick when the ball returns to the ground.

CHALLENGE

No. $(x - 3)^2 = 25$ implies $x - 3 = \pm\sqrt{25}$, so $x - 3 = 5$ or $x - 3 = -5$. Therefore, $x = 8$ or $x = -2$. The original argument omitted the case $x - 3 = -5$.

Accept equivalent equation chains when the same valid operation is applied to both sides at each step. For squared equations, accept solutions justified by $x^2 = a$ implying $x = \pm\sqrt{a}$, or by an equivalent correct case argument.